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Integrated load distribution and production planning in series-parallel multi-state systems with failure rate depending on load

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ABSTRACT

A production system containing a set of machines (also called components) arranged according to a series-parallel configuration is addressed. A set of products must be produced in lots on this production system during a specified finite planning horizon. This paper presents a method for integrating load distribution decisions, and tactical production planning considering the costs of capacity change and the costs of unused capacity. The objective is to minimize the sum of capacity change costs, unused capacity costs, setup costs, holding costs, backorder costs, and production costs. The main constraints consist in satisfying the demand for all products over the entire horizon, and in not exceeding available repair resource. The production series-parallel system is modeled as a multi-state system with binary-state components. The proposed model takes into account the dependence of machines' failure rates on their load. Universal generating function technique can be used in the optimization algorithm for evaluating the expected system production rate in each period. We show how the formulated problem can be solved by comparing the results of several multi-product lot-sizing problems with capacity associated costs. The importance of integrating load distribution decisions and production planning is illustrated through numerical examples.

1. Introduction

1.1. Motivation and problem description

Many empirical studies of mechanical systems and computer systems have proven that the workload strongly affects the failure rate (*e.g.*, [1,2]. The machines used in production systems contain mechanical and computer components. Components often work in essentially different operating modes, characterized by changing workload or environmental conditions. Those modes result in different failure rates, and lifetime distributions. The operating speeds of machinery can be set at different levels to achieve higher or lower rates of production. When machines are over-utilized and at a higher speed, more failures and interruptions are observed. In industry, it is often required to run production systems under severe conditions. For example, when the demand is too high, the machines are overloaded to avoid backorders. Managers are then faced with load distribution decisions when planning their production systems. In this context, a strong correlation between loads and failure rates may exist. This situation is observed for examples in sawmills and in manufacturing lines. It is therefore important to consider load versus failure rate relationship while performing production planning optimization. Some engineering systems are even designed to support varying amounts of loads, such as conveyers, computer processors, load-carrying systems, cutting tools, etc.

In this paper, we consider a series-parallel multi-state production system containing a set of non-identical machines (also called components). Each machine is able to support discrete loads. Each load corresponds to a possible production rate (or capacity) of the machine. The failure rate of a machine depends on its supported load. The failure rate increases in general with the load. That is, the increase of load increases the number of failures and the associated repair cost. On the one hand, with higher loads, machines have higher production rates in working states. On the other hand, the increase in load reduces the average production rate over a long horizon, because the average number of failure increases. Therefore, the expected component performance can be a non-monotonic function of its load. In [3], the authors developed a model that determines the optimal load, on each component of a series-parallel multi-state system, to provide the maximal expected performance. The objective of the

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Acronyms

LD load distribution
AFTM accelerated failure-time model
MIP mixed-integer program
MSS multi-state system
UGF universal generating function

Nomenclature

H planning horizon
T number of periods
t period index (t=1, 2, …, T)
Λ length of period t (all periods have the same length)
P set of products
p product index ($p \in P$)
d_{pt} demand of product p by the end of period t
n number of components
j component index (j=1, 2, …, n)
L_j^t load on machine j during period t
$\underline{L_j}$ maximum allowed load on component j
$\overline{\overline{L_j}}$ minimum allowed load on component j
L_{j0} baseline load of component j
λ_j^t failure rate of component j during a period t
α_j parameter of component j power law
A_j^t steady-state availability of machine j during a period t
μ_j repair rate of machine j
g_j^t production rate of component j during a period t
G_{MSS} average production rate of the MSS
h_{pt} inventory holding cost per unit of product p by the end of period t
b_{pt} backorder cost per unit of product p by the end of period t
s_{pt} fixed set-up cost of producing product p in period t
π_{pt} variable cost of producing one unit of product p in period t
u unitary cost of unused capacity
c_{jt} cost of changing load distribution for machine j from period t-1 to period t
δ_{ij} Kronecker delta function; $\delta_{ij}=1$ if $i=j$, and $\delta_{ij}=0$ otherwise
RT required repair resource
RT_0 available repair resource

Decision variables

x_{pt} quantity of product p to be produced in period t
I_{pt} inventory level of product p at the end of period t
B_{pt} backorder level of product p at the end of period t
y_{pt} binary variable, which is equal to 1 if the setup of product p occurs at the end of period t, and 0 otherwise
L^t vector of loads of all the machines for a given period t, $\mathbf{L}^t\ \{L_1^t, L_2^t, \ldots, L_n^t\}$

present paper is to integrate, for such systems, load distribution decisions, and tactical production planning considering the costs of capacity change and the costs of unused capacity. The system produces a set of products during a given planning horizon including many periods. For each product, a demand is to be satisfied at the end of period. The integrated plan should determine the quantities of items (lot-sizes) to be produced for each period, and the optimal load on each machine. The objective is to minimize the sum of setup costs, holding costs, backorder costs, production costs, capacity change costs, and unused capacity costs. The main constraints consist in satisfying the demand for all products over the entire horizon, and in not exceeding available repair resource.

An important characteristic of our model is that it takes into account the costs of changing machines capacities, and the costs of unused capacities. In practice, changing machinery nominal and calibrated production rates generates usually additional costs. That is, a load distribution plan that has less frequent changes in machine capacities will generate lower costs. Furthermore, the consequences of idle equipment may be undesirable; such left over capacity may result in a positive cost or penalty. The cost effects of idle capacity are explicitly taken into consideration (apart from the direct savings realized from not operating the system, when it is allowed to idle). We incorporate the notion of an idle capacity cost in our analysis. For simplicity, it is assumed that the total idle capacity cost is linear in the number of produced items. The cost of unused capacity is positive (a penalty). We further assume that the idle capacity cost parameter (in $/item) can be determined explicitly with a reasonable degree of accuracy.

The proposed integrated model is motivated by the fact that combining load distribution decisions with production planning may reduce the total expected cost. In fact, production planning, and load distribution planning may be in conflict. Typically, the objective of production planning is to minimize the total production cost, while the objective of load distribution planning is to maximize the system production rate. As the optimal load distribution plan tends to maximize independently the production rate, it may lead to high costs of unused capacity. Consequently, if load distribution planning and production planning activities are performed sequentially, production and load distribution plans could be not optimal with respect to the objective minimizing the combined cost. The present paper integrates load distribution decisions with tactical production planning, while taking into account the costs of unused capacity, and the costs of capacity change. The resulting integrated optimization model allows us to find the best trade-off between the different parameters of load distribution and production planning, and it considers the load versus failure rate relationship while optimizing planning of production systems.

1.2. Literature review and paper contribution

There is a substantial amount of research dealing with tactical production planning. For example, in [8,9] the authors cover the majority of the advancement in the area. Generally, production planning models tend to be deterministic optimization models designed to minimize inventory, production, and set-up costs in the planning horizon, regarding fulfillment of products demand, and machines capacities. A comparison of lot sizing methods considering capacity change costs can be found in [13]. In [14], the authors study the impact of the cost of unused capacity on production planning of manufacturing systems. In [15], the authors develop a mathematical optimization model for production lot-sizing with variable production rate and explicit idle capacity cost. In general, solution methodologies for multi-product capacitated lot-sizing problems vary from traditional linear mixed integer programming, and associated branch and bound exact methods to heuristic methods; see for example [10] for a survey.

In reliability engineering, there exist also a lot of papers dealing with optimal load distribution. One can distinguish between static and dynamic problems to consider the effects of load on component