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Testing the Goodwin growth-cycle macroeconomic dynamics in Brazil

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ABSTRACT

This paper discusses the empirical validity of Goodwin's (1967) macroeconomic model of growth with cycles by assuming that the individual income distribution of the Brazilian society is described by the Gompertz–Pareto distribution (GPD). This is formed by the combination of the Gompertz curve, representing the overwhelming majority of the population (∼99%), with the Pareto power law, representing the tiny richest part (∼1%). In line with Goodwin's original model, we identify the Gompertzian part with the workers and the Paretian component with the class of capitalists. Since the GPD parameters are obtained for each year and the Goodwin macroeconomics is a time evolving model, we use previously determined, and further extended here, Brazilian GPD parameters, as well as unemployment data, to study the time evolution of these quantities in Brazil from 1981 to 2009 by means of the Goodwin dynamics. This is done in the original Goodwin model and an extension advanced by Desai et al. (2006). As far as Brazilian data is concerned, our results show partial qualitative and quantitative agreement with both models in the studied time period, although the original one provides better data fit. Nevertheless, both models fall short of a good empirical agreement as they predict single center cycles which were not found in the data. We discuss the specific points where the Goodwin dynamics must be improved in order to provide a more realistic representation of the dynamics of economic systems.

1. Introduction

It has been noted long ago by Karl Marx that capitalist production grows on cycles of booms and busts. During a boom, profits increase and unemployment decreases since the workers are able to get better jobs and higher salaries due to shortage of manpower to feed the growing production. However, this boom is followed by a bust since less unemployment reduces the profit margin, whose recovery is achieved by a higher unemployment and a reduction of the workers' bargaining power. Smaller salaries increase the profit margin leading to renewed investment and then a new boom starts, being followed by another bust, and so on Ref. [1, Chap. 25, Section 1].

A century later Richard Goodwin [2] proposed a mathematical model which attempts to capture the essence of Marx's dynamics described above. In this model the basic dynamics of a capitalist society, as qualitatively described by Marx, is modeled by means of a modified Lotka–Volterra model where predator and prey are represented by workers and capitalists. Goodwin replaced the classic Lotka–Volterra dynamics of number of predators and preys by two new variables u and v, the former giving the workers' share of total production, which is an indirect way of describing the profit margin of capitalists, and v representing the employment rate, which is an indirect way of describing the share of those marginalized by the

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production, the unemployed workers, that is, the industrial reserve army of labor in Marx's terminology. In a boom the employment rate v increases and the workers' share u starts to increase after a time lag, meaning a decrease in profit margin. When employment rate is at its maximum this corresponds to the lowest profit margin, then the burst phase starts with a decrease in v. At this point u had already started diminishing. The essence of the model is captured as a closed orbit in the u–v phase space. Clearly these two variables are out of phase in time [3, pp. 458–464].

Although the brief description given above appears to indicate that Goodwin was able to capture Marx's observations, the model has in fact several shortcomings, the most severe one being its inability to predict quantitatively the above described dynamics (see below). The model was presented simply as an heuristic reasoning capable of giving a mathematical dressing to Marx's ideas. It was born out as a vision of the world rather than from a real-world data-inspired model in a physical sense. Despite this, or, perhaps, because of this, since its formulation Goodwin's model has attracted considerable theoretical attention in some economic circles and several variations of the original model were proposed (see Refs. [4–18] and references therein).

However, interestingly enough, almost half a century after its proposal, attempts to actually *test* this model empirically are still extremely limited. Although Goodwin's growth-cycle model is certainly influential in view of the number of *theoretical* follow-up papers cited above, studies seeking to establish its empirical soundness are limited only to Refs. [10,19–25]. This is a surprisingly short list when we consider the time span since the model's initial proposal. So little interest in empirically checking the model, especially among those who appear to have been seduced by its conceptual aspects, is even more surprising if we bear in mind that for the last 30 years or so we have been living in an era where large economic databases are easily available digitally, so large-scale checking of this model against empirical data ceased long ago to pose an insurmountable barrier. Besides, even the very few studies which actually attempted that, all point to severe empirical limitations of the model, ranging from partial qualitative acceptance to total quantitative rejection. From an econophysics viewpoint it is curious that a model with such a poor empirical record became so influential.

Despite this, the model does have some general empirical correspondence to reality on a qualitative level and this justifies further empirical studies with different databases, data handling methods and/or data type approaches. The basic aim must lie in identifying as clearly as possible where the model performs poorly in order to propose amendments and modifications. Any model, especially those theoretically seducing, can only remain of interest if it passes the test of experience, if it survives confronting its predictions with empirical data. If it does not survive this test the model must be modified, or abandoned.

This paper seeks to perform an empirical study of the Goodwin growth-cycle model using individual income data of Brazil. The study presented here was directly motivated by our previous experience in modeling Brazil's income distribution, whose results suggested a Goodwin type oscillation in the share of the two income classes detected in the data [26,27]. Building upon our previous experience with this database, we obtained yearly values of the two main variables of the Goodwin model, the labor share u and the employment rate v. Nevertheless, differently from all previous approaches for testing Goodwin's model, here the labor share was obtained by modeling the individual income distribution data with the Gompertz–Pareto distribution (GPD) and identifying u with the Gompertzian, less wealthy, part of the distribution [27]. The employment rate was also estimated from the same database, that is, from Brazil's income distribution, using the concept of *effective unemployment*.

We show that from 1981 to 2009, u and v do cycle in a form bearing similarities to what the Goodwin model predicts, that is, closed cycles. However, our results show the absence of a single cycling center and also are in complete disagreement with the ones for Brazil as reported by Ref. [25], whose analysis employed Harvie's method [22]. In addition, we attempted to see if our findings bring empirical support to the Desai–Henry–Mosley–Pemberton (DHMP) extension of the original model [9]. Our results show that this particular variation of the Goodwin dynamics has some empirical soundness, although it provides a somewhat poorer data fit as compared to the original model and also leaves three parameters to be determined by other, still unknown, means than the ones studied here, whereas the original model leaves two parameters in a similar situation. We conclude that these two models provide partial qualitative and quantitative agreement with real data, at least as far as empirical data from Brazil are concerned, but both of them, and perhaps all variations of the original Goodwin growth-cycle dynamics, require important modifications and amendments before they can be considered viable representations of the real dynamics of economic systems.

The paper is organized as follows. Section 2 presents a brief review of the original Goodwin model and its DHMP extension, focusing mostly on their dynamical equations, although some discussion about the underlying economic hypotheses and foundations of the original model is also presented. In Section 3, after a short discussion about methodology, we review the main equations behind the GPD. Section 4 analyzes the individual income data of Brazil and presents the u–v orbits in the 1981–2009 time period. Section 5 provides time variations of the employment rate as compared to workers' share so that line fittings allow us to determine some of the unknown parameters of both models. Finally, Section 6 discusses the results and presents our conclusions.

2. The Goodwin growth-cycle macro-economic dynamics

2.1. The original growth-cycle model

The model proposed by Goodwin is essentially a Lotka–Volterra predator–prey system of first order ordinary differential equations which can be written as follows [2,9,22],