Available online at www.sciencedirect.com
ScienceDirect
Mathematical and Computer Modelling 46 (2007) 892–905
MATHEMATICAL AND COMPUTER MODELLING
www.elsevier.com/locate/mcm

Analysis of benefits, opportunities, costs, and risks (BOCR) with the AHP–ANP: A critical validation☆

Diederik J.D. Wijnmalen

TNO, P.O. Box 96864, 2509 JG, The Hague, The Netherlands

Received 28 November 2006; accepted 14 March 2007

Abstract

This paper shows that the usual multiplicative synthesis of alternative priorities for benefits, opportunities, costs and risks, obtained from separate Analytic Hierarchy or Network models, can be ambiguous. The ratio of benefit and opportunity priorities to cost and risk priorities can be misleading when assessing the profitability of a project. The same holds for their additive synthesis. Both types of synthesis have been advocated in AHP/ANP literature. A quotient of these priorities with weights as coefficients, not powers, will however produce sound results, provided that the four separate models are properly related to each other by weights that make the priorities on the four factors commensurate and are obtained from magnitude comparisons. Similarly, additive synthesis with properly weighted factor priorities based on relative magnitudes will produce sound results, although use of reciprocal values of costs and risks, as often advocated in the literature, is not recommended; negative costs and risks priorities should be used instead.

Keywords: Multiple criteria analysis; Analytic hierarchy/network process; BOCR analysis; Commensurability; Weights

1. Introduction

A decision on whether or not to undertake a project usually requires investigating the positives (benefits) and negatives (costs) of that project and an attempt to express those in monetary terms such as dollars. If that project has a benefit/cost ratio >1, its benefits outweigh its costs. If there are several projects to choose among, those projects will usually be ordered according to their respective benefit/cost ratios. The ones with a ratio >1 are the attractive ones exceeding the break-even point, and the one with the highest ratio among these gives the highest return on money spent and would therefore very likely be the one to be chosen.

The problem is that often benefits and costs are difficult to express in monetary terms, especially when some of the benefits or costs are intangible, such as "improved accuracy" or "learning efforts". The Analytic Hierarchy Process (AHP) developed by Saaty [1] has been advocated as an approach that not only can deal with both tangibles and intangibles but also helps organize all aspects involved in a hierarchic structure where the benefit or cost aspects act

☆ This paper is an adapted and extended version of a paper presented at the 8th International Symposium of the Analytic Hierarchy Process, 8–10 June 2005, University of Hawai'i, Honolulu (USA).

E-mail address: diederik.wijnmalen@tno.nl.

0895-7177/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mcm.2007.03.020

as criteria and the projects as alternatives. Usually, we have separate hierarchies: one costs hierarchy and one benefits hierarchy. One has to pairwise compare the importance of cost criteria in the cost hierarchy, and the same with respect to the benefit criteria in the separate benefits hierarchy. These processes produce relative criteria weights expressed on a derived ratio scale, usually normalized to the unity sum for each family of criteria in each hierarchy. The alternative projects are pairwise compared with respect to each criterion on the lowest level of each hierarchy; their derived priorities are expressed on a ratio scale as well, again usually normalized to the unity sum per criterion. Synthesis of the alternative priorities and the criteria weights using a weighted sum produces composite alternative priorities for each hierarchy. For each alternative, its composite benefit priority is then divided by its composite cost priority. The resulting ratio value serves as a means to rank the alternatives and choose the best one, i.e. the alternative with the highest benefit/cost-priority ratio. Examples of benefit/cost analysis using the AHP were published in Saaty [1,2].

Benefit/cost analysis with the AHP has been criticized though. The main criticism is that different hierarchies produce priorities on different derived ratio scales which are usually not commensurate. The quotient of two ratio scales is again a ratio scale but has lost its clear relationship with the individual scales. The result may therefore appear meaningful as a measure of profitability, whereas it is in fact not: benefit/cost-*priority* ratios may be larger than unity when in fact the *monetary* costs exceed the *monetary* benefits. This is demonstrated in Wedley et al. [3].

Wedley et al. [4] reviewed previous literature which proposed procedures for proper benefit/cost analysis with the AHP, including suggestions by Saaty [2, page 151] to produce more meaningful ratios. Wedley et al. [3] suggested a formal magnitude adjustment procedure that converts the benefit and cost hierarchies to a common unit thus assuring that resulting benefit/cost ratios do have the desirable property of correctly indicating the break-even point. The questioning procedures they proposed are however cognitively difficult. Wedley et al. [4] proposed using linking pin methods, thereby potentially easing the questions for relating benefits to costs.

In the AHP literature thus far, only risks (R) have been added to the B/C ratio, see for example Saaty [2, 164–166]. For each alternative a $B/(C*R)$ ratio is computed based on priorities that are obtained from three different hierarchies: a benefits, a costs, and a risks hierarchy. More recent publications by Saaty [5] and Saaty and Ozdemir [6] showed that a fourth factor, opportunities (O), can be added to the analysis. This allows a full BOCR analysis using a $(B*O)/(C*R)$ ratio where positives not only include benefits but opportunities as well, and negatives not only costs but also risks. A full BOCR analysis is in some ways similar to a SWOT analysis, where not only the strong points (S) of a firm but also its (external) opportunities (O) are taken into account such as good chances of entering a new market and other favourable situations. Opportunities in BOCR analysis usually catch expectations about positive spin-off, future profits and revenue of future positive developments, whereas benefits represent current revenue or those profits from positive developments one is relatively certain of. Likewise, a firm's weak (W) points may not tell the whole story of negative aspects in SWOT analysis; external threats (T) concerning competition or unfavourable developments in society must be dealt with as well. Risks in BOCR analysis are supposed to catch the expected consequences of future negative developments, whereas costs represent (current) losses and efforts and consequences of negative developments one is relatively certain of. BOCR analysis enables therefore a potentially richer analysis than a mere BC analysis, although many of the aspects that define the factors and their relationships are usually difficult to specify and quantify.

Recently, the Analytic Network Process with supporting software was developed by Saaty [5], enabling one to model systems with feedback and dependence. Especially in the context of the ANP, many applications of BOCR analysis are offered where a network model with one or more networks for each of the four BOCR factors is set up instead of a hierarchy. This allows for modelling interrelationships between the elements defining each of the four overall factors.

Not only B/C analysis but also $B/(C * R)$ analysis has been criticized, for example by Millet and Wedley [7]. These authors argue that the product of costs and risks is not meaningful or justified, with the added argument that differences in relative importance are not accounted for. Similar arguments may hold for BOCR analysis as well. Furthermore, it has been argued (Wedley [8]) that opportunity and risk priorities could be regarded as probabilities. Higher opportunity and risk priorities would then represent greater opportunity or risk likelihoods, and their products with benefit and cost priorities, respectively, would produce expected benefit and expected cost values. This paper does not enter into this discussion, although it does, for other reasons that will become clear later on, criticise the quotient of products as proposed by Saaty [5].

This paper will not question a BOCR analysis per se, nor will it address the way in which composite alternative priorities on each of the four merit factors (B, O, C, R) are computed, be it using AHP-based or ANP-based models. It is rather the computational method for getting meaningful synthesized results that is the paper's subject. It therefore