Available online at www.sciencedirect.com
ScienceDirect
Mathematical and Computer Modelling 46 (2007) 860–891
MATHEMATICAL AND COMPUTER MODELLING
www.elsevier.com/locate/mcm

Time dependent decision-making; dynamic priorities in the AHP/ANP: Generalizing from points to functions and from real to complex variables

Thomas L. Saaty

University of Pittsburgh, Pittsburgh, PA 15260, United States

Received 28 November 2006; accepted 14 March 2007

Abstract

Because good decisions depend on the conditions of the future, and because conditions vary over time, to make a good decision requires judgments of what is more likely or more preferred over different time periods. There are at least three ways to deal with dynamic decisions. One is to include in the structure itself different factors that indicate change in time such as scenarios and different time periods and then carry out paired comparisons with respect to the time periods using the fundamental scale of the AHP. The second is to do paired comparisons as rates of relative change with respect to time. This is done at different points of time as representatives of intervals to which they belong. These intervals can have different lengths. For each representative point one needs to provide a pairwise judgment about the relative rate of change of one alternative over another and derive mathematical functions for that matrix of comparisons for one of the time periods. The third is to do what I proposed in my first book on the AHP and that is to use functions for the paired comparisons and derive functions from them. It is usually difficult to know what functions to use and what they mean. Ideas and examples are presented towards the development of a theory for dynamic decision making.

Keywords: Dynamic priorities; Time dependent AHP; AHP; ANP; Priority functions; Time horizons; Neural firing

1. Introduction

The Analytic Hierarchy Process (AHP) for decision-making is a theory of relative measurement based on paired comparisons used to derive normalized absolute scales of numbers whose elements are then used as priorities [1,2]. Matrices of pairwise comparisons are formed either by providing judgments to estimate dominance using absolute numbers from the 1 to 9 fundamental scale of the AHP, or by directly constructing the pairwise dominance ratios using actual measurements. The AHP can be applied to both tangible and intangible criteria based on the judgments of knowledgeable and expert people, although how to get measures for intangibles is its main concern. The weighting and adding synthesis process applied in the hierarchical structure of the AHP combines multidimensional scales of measurement into a single “uni-dimensional” scale of priorities. In the end we must fit our entire world experience into our system of priorities if we are going to understand it.

E-mail address: saaty@katz.pitt.edu.

0895-7177/$ - see front matter
doi:10.1016/j.mcm.2007.03.028

Sound decision making not only requires that we look ahead as a process of thinking and planning, but also that our structures be flexible to include change so that we can constantly revise our decision because the system itself is in imbalance resulting in a non-stationary optimum. Thus optimizing inefficiently by selecting a best decision at a given time can only be a sub-optimum over a long range time horizon because of the fundamental influence of change in any survivable system. There are three ways to cope with this problem. The first is to make the best choice according to short-, mid- and long-term merits with a feedback loop or a holarchy used to prioritize them according to their benefits, opportunities, costs and risks evaluated in terms of strategic criteria used in general to guide all our decisions [3]. The other is to make the judgments mathematically depend on time and express them as functions, discussed in this paper. The third and less practical way is to revise a decision every once in while. Although many decisions can be tested in advance through sensitivity analysis to determine their long-term stability, revision after implementation can be both controversial and costly. One may have to abandon an already finished resource intensive alternative for another such costly alternative. In politics, we circumvent doing it by electing new leaders with the perspective we want, so they can focus on making better decisions for pressing needs in society.

So far most of us have had no way to combine dollars with yards or pounds to trade them off. We would be truly multi-dimensional if we could combine the different dimensions into a single dimension that represents our priority of importance. That is precisely what the AHP and ANP help us do in a more or less precise way, depending on the level of experience that we bring to bear on a decision problem. Until recently, the AHP and ANP have been *static* in that they have used numbers and derived numbers to represent priorities. What we need is to make them dynamic by using numbers or functions and then deriving either numbers that represent functions like expected values, or deriving functions directly to represent priorities over time. My aim here is to extend the AHP/ANP to deal with time dependent priorities; we may refer to them as DHP/DNP (Dynamic Hierarchy Process/Dynamic Network Process). At this point we may not know enough to develop the necessary fundamental scale of functions to use in making paired comparisons of intangibles. But if nothing else DHP and DNP work with tangibles now and we need to weight and trade off tangibles as functions of time.

Time dependent decision-making that we call dynamic decision-making is a subject that we need today. So far we have thought of our decisions as known alternatives to choose from. But these alternatives may evolve over time along with our preferences for them like stocks in the stock market whose prices constantly change over time. Our actions need to vary over time like a medicine capsule that releases different amounts of chemical at different times. Time dependent decisions are a reality and not a complicated idea that we can ignore. At a minimum they are needed in technical design problems in which the influences of several tangible design factors change over time and tradeoffs must be made among them to enable the system to respond differently and continuously over the time of its operation. But the power and potential of the subject lie in its use of judgment to make comparisons to derive relative real valued functions for intangibles from paired comparisons. Because we can do that for real numbers we can also do it for complex numbers. They have a modulus (magnitude) and an argument (direction), each of which is real. That is where we need to go later to derive relative complex functions from paired comparison expert judgments. The modulus is estimated in the usual way of comparing magnitudes and the argument or angle is estimated along the lines of rate of change comparison given below. In this way the two parts of a complex number are derived from paired comparisons.

There are essentially two analytic ways to study dynamic decisions: structural, by including scenarios and time periods as elements in the structure that represents a decision, and functional by explicitly involving time in the judgment process. A possible third way would be a hybrid of these two.

The *structural* method is most familiar today and it involves using scenarios or time periods as factors in the hierarchic or network structure of a decision, and then making appropriate judgments. Generally contrast scenarios such as optimistic, status quo and pessimistic, or more specific ones such as different values for the economy or the stock market are put near or at the top of a hierarchy. The likelihood of the scenarios is first determined in terms of higher level criteria under the goal such as economic, political, social and technological, that are themselves prioritized according to their prevailing influences over a certain time horizon. Judgments are provided for the behavior of the alternatives with respect to the factors encountered under each scenario [4]. Synthesis reveals the best alternative to follow in view of the mix of potential scenarios. For more detail about this method see my book on Analytical Planning where contrast scenarios are discussed. The other structural method is to put actual time periods at the "bottom" of the structure, prioritize them and finally combine their priorities by, for example, using the idea of expected value. This method was used in estimating when the US economy would recover and is illustrated in the next section.