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Validation examples of the Analytic Hierarchy Process and Analytic Network Process

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Abstract

One way to validate a scientific theory is to show that the results predicted by the theory give correct answers; that is, that they match known results. In the Analytic Hierarchy Process (AHP) this usually means finding examples with measures in an already known scale. To validate AHP priority vectors against measures from known scales the measures must first be normalized by dividing by their sum. When the two vectors are the same, or close, then one can say the results of the AHP model have been validated. The AHP and its generalization, the Analytic Network Process (ANP), can be validated at several levels ranging from priority vectors derived from pairwise comparison matrices to the synthesized priorities for a hierarchical model, to the priorities derived for the elements in an ANP network from the limiting supermatrix (perhaps most impressively validated by estimating market share of companies using intangible factors), to the overall results from complex ANP models involving several levels of networks. Many validation examples are presented along with a discussion of the compatibility index that can be used to measure closeness of priority vectors.

Keywords: Validating the Analytic Hierarchy Process; Validating the Analytic Network Process; Compatibility index for the AHP; Validation examples for the AHP; Validation examples for the ANP

1. Introduction

"Scientific truth, that is to say the validity of an accepted theory, depends on two important kinds of factors: the guiding principles ... and what we have called the process of empirical verification ... these two factors are crucial in the establishment of any theory relating to any kind of knowledge".

According to LeShan and Marganau [1], quoted above, scientific theories must be verified against things that happen in the real world and must follow the guiding principles of science that have been loosely enumerated as *simplicity*, *extensibility*, *multiple connections*, *logical fertility*, *stability of interpretation*, *causality*, and *elegance*.

In science validation is a central concern. It is as relevant in the social sciences, although more difficult to do, as it is in the physical sciences. Validating subjective observations made by experts or by experienced people with their

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perceptions is what we intend to deal with here. A decision theory that hides behind subjectivity and a claim to being normative (i.e., the theory specifies what is good for you) has no way of being validated even when it claims success. One cannot help but be struck by the rather precise way in which people using AHP are able to understand influences that they have internalized over a period of time. They can capture the importance of these influences by making pairwise comparison judgments expressed using the AHP fundamental scale in a matrix which results in a priority vector or in a hierarchical AHP model composed of a number of pairwise comparison matrices. Priority vectors can be compared to measures that have been transformed into relative form — that is, they have been normalized so that they sum to 1.

Rational decision-making is a talent we must encourage if we want to be more effective in implementing our ideas in the real world with its risks and resistance to change. There are two kinds of decisions. One is to determine what we prefer the most, known as *normative* decision making and the other is how to make a best choice given all the influences in the world around us that can affect the optimality of any choice we make, known as *descriptive* decision making. When we think about normative decisions it is easy to see why we do not wish anything to happen that can undermine the best choice we make and thus if we choose an alternative, we do not want it to be influenced by other alternatives that arrive (or occur to us) later. That is why preserving rank when a new alternative is added or an old one deleted is essential for this kind of thinking. Descriptive statements are falsifiable statements that attempt to describe the real world as it is. Normative statements legislate how things ought to be, what is good and what is bad; what is right and what is wrong. Normative statements can never be proven to be correct and workable, but only disproved with examples of what they recommended failing. A useful way to show their falsehood is by pointing to real life occurrences that they violate in spite of their claims.

But in reality how good any choice we make depends on how well we know our alternatives as compared with each other and with others outside the collection being compared so we can rank them as to how good they are. The drawback is that our knowledge of the alternatives may be very limited. It is known in practice that new alternatives can influence what we thought of in ranking other alternatives one by one earlier. Yet normative theories want to overlook that in all situations. A descriptive approach to decision making must cover both the normative and the descriptive ways of thinking. Such a theory must allow for enforcing the stability of choices in certain decisions and for their potential variability in others.

Decision-making involves prioritizing our ideas according to the circumstances we face now or might face in the future. A fundamental problem in decision-making is how to measure intangible criteria and how to interpret measurements of tangibles correctly so they can be combined with those of intangibles to yield sensible, not arbitrary numerical results. A crucial test is whether actual measurements can be used precisely as they are when needed.

The AHP/ANP is fundamentally a way to measure intangible factors by using pairwise comparisons with judgments that represent the dominance of one element over another with respect to a property that they share. Many examples are worked out by knowledgeable people without entering all the judgments but only contrasting ones that form a spanning tree which covers all the elements thus shortening the time in which the exercise is done. The AHP/ANP has found useful applications in decision making which involves numerous intangibles. It is a process of laying out a structure of all the essential factors that influence the outcome of a decision. Numerical pairwise comparison judgments are then elicited to express people's understanding of the importance, preference or likely influence of these elements on the final outcome obtained by synthesizing the priorities derived from different sets of pairwise comparisons. Sensitivity analysis is performed in the end to determine the stability of the outcome to wide perturbations in the judgments. The process has been validated in practice in many ways as we show in the next section.

The AHP uses additive synthesis which means multiplying the priorities of the alternatives by those of their respective criteria and adding. It was once proposed by F. Lootsma of Delft that it would be better to use multiplicative synthesis by raising the alternative priorities to the power of their respective criterion priorities and multiplying the resulting priorities of the alternatives. He had a few advocates who published on it, but no one has promoted this idea since T. Saaty brought to the attention of Lootsma and others several years ago that an alternative with the same priority (value less than one) under two criteria when raised to the respective powers of the criteria would result in the alternative under the larger priority criterion receiving a smaller value — contradicting common sense. Consider for example raising the fraction $1/2$ to the power of .7 (which yields .6156) and then to the power of .3 (which yields .8123). Because of this multiplicative synthesis is not a viable option for synthesis.