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A geometrical framework for solving sunlighting problems within CAD systems

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Abstract

We propose a framework based on integral geometry that makes it possible to express and solve most geometrical sunlighting problems within CAD systems: the direct problems for assessing sunlighting in a scene and the inverse ones for achieving sunlighting constraints in a design context. The heart of our method is the sunlighting volume that we denote $\Pi(P, T)$. We describe this notion and we show how to implement it in an architectural and urban design process.

Keywords: Solar geometry; Inverse problems; CAD system; Architectural design

1. Introduction

We distinguish the *direct* sunlighting problems from the *inverse* ones. The former consist of determining the sunlighting states of a given scene using both shadows and shadings; the latter consist of resolving the geometrical conditions that enable a scene to achieve a given sunlighting constraint in a design context, i.e. sunlighting becomes a formgiver [1]. Much work has been done in both directions. Thus, numerous evaluative systems are proposed to assess solar properties of architectural and urban environments [2,3]. Moreover, some researchers have developed generative approaches for optimising solar devices and for expressing solar design intents [4–7]. However, despite much effort, the integration of these tools in existing geometrical CAD systems remains limited. Besides, few of the proposed systems are capable of solving both the direct and inverse problems of sunlighting within the same conceptual framework.

In this paper, we propose a geometrical framework based on integral geometry [8] for managing sunlighting problems. This framework offers three main benefits: it gives homogeneous expressions and solutions for the direct sunlighting problems of shadings and shadows; it enables the generalisation of these problems either in space or time and it offers solutions to the inverse problems of sunlighting.

The heart of our framework is the sunlighting volume that we denote $\Pi(P, T)$. We define this notion in Section 2 within a set of geometrical notions that make-up our method. In Sections 3 and 4, we give solutions to the direct and inverse problems. Finally, we show how these methods can be implemented in an existing CAD system (Section 5), and why they offer a powerful framework to cope with sunlighting problems in architectural and urban design (Section 6).

2. The integral geometry of sunlighting

Let Σ be a geometrical scene located at the latitude λ. We denote p any point of Σ and P any continuous set of points in Σ. We consider D_λ the set of all the apparent solar directions at the latitude λ, and we denote t any direction included into D_λ (i.e. any instant of the solar year). We define a time period T as any continuous set of instants t, that is consequently a composition of intervals of instants among days and months. T can likewise be seen as a geometrical patch of D_λ. Finally, we define the sunbeam $R(p, t)$ as the half line starting from p in the direction of t. We define the inverse sunbeam $R(p, -t)$ starting from p in the opposite direction of t (which is the perceived natural sunbeam).

2.1. Sunlighting volumes

Let us now consider sunlighting volumes settled as continuous sets of sunbeams. First, we define the sunlighting pyramid $\pi(p, T)$, that is the set of sunbeams starting at the point p for all the instants t of the time period T: $\pi(p, T) = \{R(p, t), t \in T\}$. In the same way, we define the sunlighting prism $\varpi(P, t)$, that is the set of sunbeams defined for

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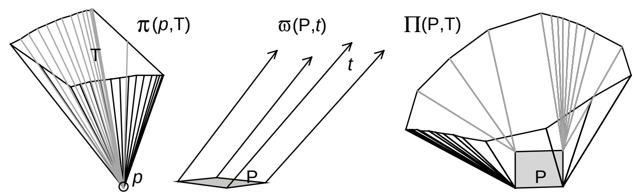

Fig. 1. The three kinds of sunlighting volumes: the sunlighting pyramid defined for a point p and a set of instants T (left); the sunlighting prism defined for a set of points P and a single instant t (centre), and the sunlighting volume defined for a set of points P and a set of instants T (right).

all the points p of P and for the single instant t: $\varpi(P, t) = \{R(p, t), p \in P\}$. Finally, we define the complex sunlighting volume $\Pi(P, T)$ as the set of sunbeams starting from all the points p of the set P for all the directions t of the time period T, that is: $\Pi(P, T) = \{R(p, t), p \in P, t \in T\}$. Note that $\Pi(P, T)$ can be equally considered as the union of simple sunlighting pyramids $\pi(p, T)$ when p draws P, or as the union of prisms $\varpi(P, t)$ when t draws T, that is (Fig. 1)

$$\Pi(P, T) = \bigcup_{p \in P} \pi(p, T) = \bigcup_{t \in T} \varpi(P, t) \tag{1}$$

2.2. Cores of sunlighting volumes (solar envelopes)

We define $\Delta\Pi(P, T)$ the core of $\Pi(P, T)$ as the intersection of the sunlighting prisms $\varpi(P, t)$ when t draws T, that is (Fig. 2)

$$\Delta\Pi(P, T) = \bigcap_{t \in T} \varpi(P, t) \tag{2}$$

The core $\Delta\Pi$ embodies the set of all the points of the scene Σ that are affected by all the sunbeams defined by all points of P during all instants of T. In other words, any point of Σ within $\Delta\Pi$ will have its shadow within P whatever is the instant of T. This means that the shadows of any volume built in $\Delta\Pi$ will never spill over the edge of P during T (Fig. 3). This is the definition of Knowles' solar envelope as a container to regulate development within limits derived from the sun's relative motion [4,5]. We offer here a geometrical framework for defining and using this notion usually handled with empirical geometrical methods [7].

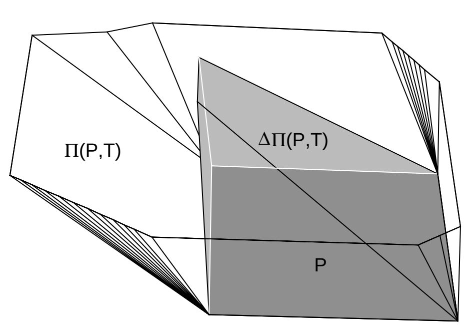

Fig. 2. The core of a sunlighting volume.

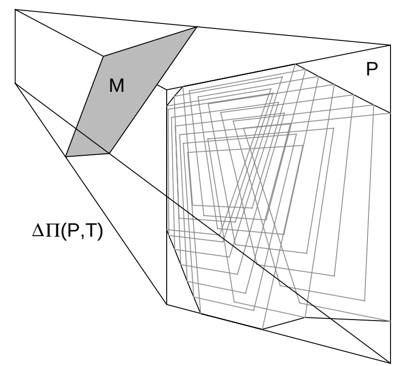

Fig. 3. M defined in the core of a sunlighting volume $\Pi(P, T)$ has its shadows within P whatever the instant of T is.

Remark that the shape of the core is related to the shape of the associated sunlighting volume. The more the volume is open, i.e. the larger the temporal period, the less high is the core. The two extreme cases are the whole solar year for which the core $\Delta\Pi(P, T)$ is reduced to the base P, and each instant t, for which the core $\Delta\Pi(P, T)$ is equal to $\Pi(P, T)$, i.e. in that case, the sunlighting prism $\varpi(P, t)$.

3. Direct sunlighting problems

Direct problems of shadows and shadings can be easily expressed, generalised in either time or space and geometrically solved using integral geometry.

3.1. Generalised shadows

Let $\Omega(P, t)$ be the shadow of P in the scene Σ for the instant t. By definition, $\Omega(P, t)$ is the intersection between the sunlighting prism $\varpi(P, -t)$ and the scene Σ, that is: $\Omega(P, t) = \varpi(P, -t) \cap \Sigma$. If we consider the time period T instead of the instant t, we can express the generalised shadow of P during T, that is the union of all the shadows of P for all the instants t of T

$$\Omega(P, T) = \bigcup_{t \in T} \Omega(P, t) = \bigcup_{t \in T} (\varpi(P, -t) \cap \Sigma) = \Pi(P, -T) \cap \Sigma \tag{3}$$

Therefore, the generalised shadow of P during T is given by the intersection between the sunlighting volume $\Pi(P, -T)$ and the objects of the scene Σ. This is the exact set of points of Σ that are exposed to the shadow of P during at least one instant of T. In the same way, we prove that the exact set of points of Σ that are exposed to the shadow of P during all the instants of T are those that are in the core of the sunlighting volume $\Delta\Pi(P, -T)$. Then we denote the core of shadow: $\Delta\Omega(P, T) = \partial\Pi(P, -T) \cap \Sigma$ (Fig. 4).

Obviously, if P is an opening instead of a shading, then the generalised shadow is the union of all sunspots generated by P during T (generalised sunspot) and the core of shadow