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RESEARCH PAPER

Cost-time Minimization in a Transportation Problem with Fuzzy Parameters: A Case Study

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Abstract: In real world applications the supply, the demand and the transportation cost per unit of the quantities in a transportation problem are hardly specified precisely because of the changing economic and environmental conditions. It is also important that the time required for transportation should be minimum. In this paper a method has been proposed for the minimization of transportation cost as well as time of transportation when the demand, supply and transportation cost per unit of the quantities are fuzzy. The problem is modeled as multi objective linear programming problem with imprecise parameters. Fuzzy parametric programming has been used to handle impreciseness and the resulting multi objective problem has been solved by prioritized goal programming approach. A case study has been made using the proposed approach.

Key Words: transportation economy; impreciseness; fuzzy parametric programming; multiple criteria optimization; goal programming

1 Introduction

The problem of minimizing the total cost of transportation has been studied since long and is well known. In a time minimizing transportation problem, the time of transporting goods is minimized to satisfy certain conditions in respect of availabilities at sources and requirements at destinations. The basic difference between cost minimizing and time minimizing transportation problem is that the cost of transportation changes with variations in the quantity but the time involved remains unchanged and irrespective of the quantities. The time minimizing transportation problem has been studied by Hammer[1,2], Garfinkel and Rao[3], Szware[4], Bhatia, Swarup and Puri[5], Ramakrishnan[6], Sharma and Swarup[7], Seshan and Tikekar[8] and by several other authors. Time-cost trade off means the problem of minimizing the transportation cost in addition to minimizing the time of the transportation. Time-cost trade off analysis has been discussed by Satya Prakash[9], Bhatia, Swarup and Puri[10] and several other authors. Satya Prakash[9] has used goal programming approach to solve the problem. Liu[11] discussed a method for solving the cost minimization transportation problem with varying demand and supply.

Most of the models developed for solving the transportation problem are with the assumption that the supply, demand and the cost per unit values are exactly known. But in real world applications, the supply, the demand and the cost per unit of the quantities are generally not specified precisely i.e. the parameters are fuzzy in nature. Impreciseness in the parameters means the information for these parameters are not complete. But even with incomplete information, the model user is normally able to give a realistic interval for the parameters. Carlsson and Korhonen[12] and Chanas[13] discussed parametric approach to deal with the fuzzy parameters.

In this paper cost and time minimization transportation problem has been dealt when the supply, the demand and the transportation cost per unit of the quantities are fuzzy. Impreciseness in the parameters has been dealt with using the concept developed in Ref. [12] and for the time minimization, the set of times (required for different sources to different destinations) is partitioned suitably. Linear and exponential membership functions have been considered for the imprecise parameters. The problem has been posed as multi objective linear programming problem and solved using preemptive goal programming approach by assigning different priorities

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to the different objectives. The model intends to obtain a compromise solution with minimum transportation cost as well as minimum transportation time. A case study has been made for the transportation of coal from different collieries of a regional coal company situated at Jharia coalfield (India) to different washeries.

2 Formulation of the problem

Suppose that there are m origins and n destinations. It is assumed that the quantities available at the origins, demand at the destinations and the cost per unit of the quantity are not given precisely. However, a realistic interval is available for each of these imprecise parameters. Further it is assumed that it is possible to transport the product from any origin to any destination and the time of the transportation does not depend on the amount of product transported. Let t_{ij} be the units of time of transportation of the product from origin i to the destination j, c_{ij} be the cost of transportation of one unit of the product from origin i to the destination j, a_i be the units of the product available at origin i, b_j be the demand at the destination j and x_{ij} be the number of units of the product transported from origin i to the destination j. The primary objective is to minimize the total cost of transportation and the secondary objective is to minimize the duration of the transportation subject to the constraints.

The fuzzy mathematical formulation of this bi-objective problem may be presented as follows:

Find $x_{ij}\geq 0$ (i =1, 2, ···, m; j =1, 2, ···, n) to

$$\text{Minimize } Z_1=\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij} \quad (1)$$

$$\text{Minimize } Z_2=\max\{t_{ij} : x_{ij}>0\ (i=1, 2, \cdots, m; j=1, 2, \cdots, n)\} \quad (2)$$

Subject to the constraints:

$$\sum_{j=1}^{n} x_{ij} \leq \tilde{a}_i \qquad (i=1, \cdots, m) \quad (3)$$

$$\sum_{i=1}^{m} x_{ij} \geq \tilde{b}_j \qquad (j=1, \cdots, n) \quad (4)$$

$x_{ij}\geq 0$ (i=1, 2, ···, m; j=1, 2, ···, n), where $\tilde{c}_{ij}$, $\tilde{a}_i$, $\tilde{b}_j$ are the imprecise parameters for the transportation cost per unit of quantity, availability at source and demand at the destinations. The model applies when the total availability exceeds the total demand.

These imprecise parameters may be represented by the intervals $[c_{ij}^0, c_{ij}^1)$, $[a_i^0, a_i^1)$ and $[b_j^0, b_j^1)$ respectively. In each of the intervals the first component with subscript 0 represents the "risk free" and the second component with subscript 1 represents the "unrealistic/impossible" parameters[12]. It is clear that the solution with lower bounds (first component) should most certainly be implementable and the solution with upper bound (second component) is practically not implementable. Thus, when we move from "risk free" towards "impossible" parameter values, we move from solutions with a high grade to solutions with a low grade on implementing. i.e., from "secure" to "optimistic" solutions. It is required to find an optimal compromise solution as a function of the grades of imprecision in the parameters.

3 Solution procedure

There are many possible forms for membership functions: linear, exponential, hyperbolic, hyperbolic inverse, piece-wise linear, etc. The linear form of membership function is most common and suitable to describe impreciseness in many real world problems. Exponential form of membership functions is another suitable form, which is not as restrictive as the linear form, but flexible enough to describe the grades of precision in the parameter values. Unlike linear membership function, for nonlinear membership functions the marginal rate of increase (or decrease) of membership values as a function of model parameters is not constant. Slowinski[14] have used exponential membership function for the fuzzy multi criteria linear programming problem. Gupta et al.[15] studied a pair of fuzzy primal-dual linear programming problems and calculated duality results using an aspiration level approach using exponential membership function. For the above fuzzy bi-objective problem, suitable membership functions may be chosen to match the reality in the field. Based on the analysis of data, we have considered the linear membership function for the imprecise availability and the exponential membership function for the imprecise cost and the demand parameters.

3.1 Multi objective model

Let the membership functions for cost, availability and demand imprecise parameters are denoted by $\mu_{c_{ij}}, \mu_{a_i}$ and μ_{b_j}, respectively.

We will use the subscript p to refer to a parameter of the model (1)-(4). Let us scale the membership functions such that μ_p=1 if $p\leq p^0$ and μ_p=0 if $p\geq p^1$.

The linear membership function (Fig. 1) considered for the availability, may be defined as[16]:

$$\mu_p = \begin{cases} 1 & \text{if } p \leq p^0 \\ \dfrac{p - p^1}{p^0 - p^1} & \text{if } p^0 < p < p^1 \\ 0 & \text{if } p \geq p^1 \end{cases} \quad (5)$$

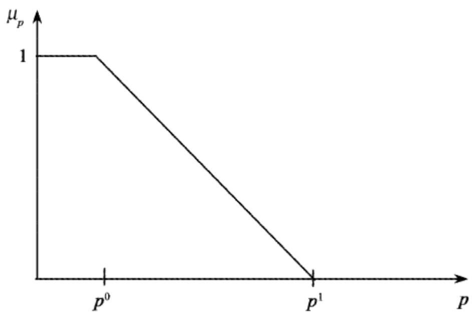

Fig. 1 Linear membership functions