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Detecting approximate symmetries of discrete point subsets[☆]

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Abstract

Detecting approximate symmetries of parts of a model is important when attempting to determine the geometrical design intent of approximate boundary-representation (B-rep) solid models produced e.g. by reverse engineering systems. For example, such detected symmetries may be enforced *exactly* on the model to improve its shape, to simplify its analysis, or to constrain it during editing. We give an algorithm to detect *local approximate symmetries* in a discrete point set derived from a B-rep model: the output comprises the model's potential local symmetries at various *automatically* detected tolerance levels. Non-trivial symmetries of subsets of the point set are found as *unambiguous permutation cycles*, i.e. vertices of an approximately regular polygon or an anti-prism, which are sufficiently separate from other points in the point set. The symmetries are detected using a rigorous, tolerance-controlled, incremental approach, which expands symmetry seed sets by one point at a time. Our symmetry cycle detection approach only depends on inter-point distances. The algorithm takes time $O(n^4)$ where n is the number of input points. Results produced by our algorithm are demonstrated using a variety of examples.
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1. Introduction

Many manufactured objects exhibit global and local symmetries as a feature of their design or function, or for ease of manufacturing or analysis [1]. Furthermore, symmetry is also common in natural shapes [2] and designers prefer symmetrical shapes for reasons of aesthetics and simplicity [3]. This is particularly true for engineering objects conventionally represented by boundary-representation (B-rep) models, such as the one shown in Fig. 1.

While such symmetries *may* be explicitly represented along with a B-rep model, often they are not explicitly given, for example, where a model has been created by reverse engineering, or where a model has been transferred from one CAD system into another. Furthermore, in cases like these, the symmetries are often not *exactly* present, but only approximately present, due to measurement errors in the scanning process, and approximation and numerical errors in model reconstruction during reverse engineering [4]. Different CAD systems often use different tolerances [5], and what is symmetric in one CAD system may not be symmetric in another.

Explicit detection of symmetries in such geometrical models has many potential uses: for example, to improve the shape of a model by enforcing intended symmetries, to enable faster analysis, to place constraints on editing operations, and so on. We are thus interested in detecting the symmetries intended by a designer in a B-rep model, but which are only approximately present.

Our previous methods for geometric design intent detection can detect *global* approximate symmetries [6], approximate *congruencies* between sub-parts [7], and other local *regularities*, e.g. parallel and orthogonal planes [8]. This paper considers a different issue not solved by such approaches: finding *local* approximate symmetries in a B-rep model. For example, the model in Fig. 1 has cylindrical holes arranged with an eight-fold rotational symmetry, and slots with a sixteen-fold rotational symmetry.

To detect local symmetries, we use similar ideas to those used for global symmetric detection in [6,7]. As in these papers, we extract *characteristic points* from a solid model, which when used with connectivity and face type information, are sufficient to determine its symmetry, should symmetry

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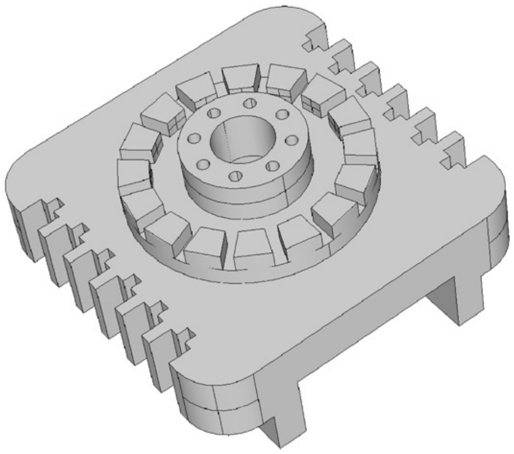

Fig. 1. A B-rep model with many local symmetries.

be present. Essentially, these points are the vertices of the B-rep model, together with other special points needed to characterize curved edges and faces. For example, a straight line is characterized by its two end points, whereas for an edge known to be a circular arc, using one other point taken to be the mid-point of the arc is sufficient to both fix its radius, and to determine which of the two arcs between those end points we want. A discussion of how to select these characteristic points is given in [7].

Note, then, that we start from a very different point of view to symmetry approaches used in image processing, e.g. [9–12], and mesh processing [13–15]. These are designed to work on dense point data, where the point distributions are far more important than locations of individual points. Generally, their aim is to detect one or a few *dominant* approximate symmetries by partial matching of images or meshes under user selected tolerances. In contrast, we wish to find symmetries in B-rep models that are intended to be exact, but are approximate due to their origins. We furthermore wish to generate *all* possible subset symmetries, where any one may belong to quite a small part of the model (such as a hole, or row of slots in a complex model). We use as a basis a carefully selected and generated point set from a B-rep model, not a dense point set covering the whole surface of the B-rep model. Our algorithm thus processes far fewer points than a mesh symmetry algorithm; in our algorithm, both the position, and existence, of every individual point is significant. However, we speculate that it might be possible to apply our method to detecting symmetries of meshes *if* a suitable means could be found for defining and extracting carefully chosen key feature points.

In summary, the main novel contribution of the paper is a rigorous definition of and an algorithm to detect *local approximate symmetries* possessed by subsets of a set of points in 2D or 3D. By letting this set of points be carefully chosen characteristic points extracted from a B-rep model as described above, these approximate point subset symmetries in turn directly correspond to approximate local symmetries of an approximate B-rep model. Finding approximately symmetrical subsets of a point set is an important topic not addressed by previous work. Here we are considering points which the symmetry maps in a one-to-one fashion onto each other. Mitra et al. [15] have considered the different problem of approximate maps of dense point clouds representing part of the surface of an object onto other dense point clouds from the same object—but these are not pointwise maps. Other related work is later discussed in Section 2.

The detected symmetries include rotational symmetries and rotation-reflection symmetries, i.e. vertices of an approximately regular polygon or an anti-prism. Each symmetry is represented as an *unambiguous (permutation) cycle* on that subset of points. Each *symmetry* corresponds to a transformation which maps a subset of the point set onto itself. As we assume that the input model is *approximate*, the subset may only map approximately onto itself under the symmetry. A *(permutation) cycle* is a subset of a permutation whose elements trade places with one another. It describes the orbit of a single point under consecutive application of a symmetry transformation (in the exact case), specifically under rotation and rotation-reflection. By *unambiguous* cycles we mean that the points involved in the subset are sufficiently far away from other points in the input point set, so that there is no chance of confusion as to which point maps to which under the symmetry transformation, given its approximate nature. These ideas are explained more carefully and rigorously in Section 3.

Note that in this paper we *only* concern ourselves with finding each individual cycle *separately*. Thus, given a regular prism, we will output one cycle corresponding to the vertices at one end, and a separate cycle for the vertices at the other end—even though (at least in the exact case) these two point subsets share the *same* symmetry transformation. Clearly, extracting higher level information is important: *merging* the cycles found by the method given in this paper will be addressed in future work. In the following, we shall always mean a cycle when we talk about a symmetry, unless we say otherwise.

Before we go further, we should just mention a special case. Clearly, every *pair* of points trivially defines an *exact* two-fold rotational symmetry, a reflection symmetry, and an inversion symmetry cycle. These cycles can be trivially ‘found’ by simply enumerating every pair of points, and so are not further discussed here. Thus, we consider how to find rotational symmetry cycles, i.e. vertices of an approximately regular polygon, and rotation-reflection symmetries, where the symmetry transformation comprises reflection in a plane followed by rotation about an axis perpendicular to that plane, i.e. vertices of an anti-prism.

There are seven elementary symmetry transformations: reflection, inversion, translation, rotation, glide reflection, rotation reflection and screw translation [16]. However, discounting inversion and reflection, only two other kinds – rotation and rotation-reflection – have finite cycles (i.e. if we apply the symmetry operation enough times, the points go back to their original permutation). Translational symmetries, and glide reflections and screw translations, which are combinations of translation respectively with reflection and rotation, must always be incomplete for finitely many points. Handling incomplete symmetries, both of this kind, and e.g. incomplete

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