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Constrained fitting for 2D profile-based reverse modeling

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Abstract

Profile curve reconstruction is crucial to surface reconstruction in reverse engineering. In this paper, we present a new constrained fitting method involving lines, circular arcs and B-spline curves for profile curve reconstruction. By using similarity transformation, we reduce the condition number of the Hessian matrix involved in the optimization process and, therefore, the numerical stability is significantly improved. Several industrial examples are presented to demonstrate the efficiency of our method. This paper describes a 2D constrained fitting method for profile curve reconstruction in reverse engineering. The method is an extension to the published methods for 2D constrained fitting. Further more, the numerical problem associated with constrained fitting is tackled in our paper. The described method has been implemented in RE-SOFT, which is a feature-based reverse engineering software developed by the CAD/CAE/CAM Lab of Zhejiang University.

Keywords: Constrained fitting; Reverse engineering; Similarity transformation; 2D-based modeling; Curve reconstruction

1. Introduction

With the extensive use of modern CAD/CAE/CAM technologies, both quality and efficiency of design, analysis and manufacture are improved during the last two decades. In any case, computer models of objects must be available before the advantages of these technologies can be exploited. Unfortunately, computer models (here we mean geometric models) of objects are not always available for some reasons. Reverse engineering technology, which converts existing objects to computer models, is developed to deal with such a situation [1].

Surface reconstruction, as one of the most important step in reverse engineering process, has received extensive research in literature. Pratt [2] uses algebraic distances to fit n-dimensional (typically n is 2 or 3) algebraic surfaces. He achieves great improvements in circle and sphere fitting with suitable quadratic normalizations, which make algebraic distances good approximations to geometric distances. Marshall et al. [3] present the least squares fitting method of conventional analytical surfaces (sphere, cylinder, cone and tori), in which faithful approximations to geometric distances are used. With such distance metrics, good fitting results are acquired with reasonable computational costs. Weiss et al. [4] consider the problem of parametric freeform surface fitting to unorganized points, in which three subproblems, i.e. the number and placement of knots, the weights of the smoothing functionals and the best parameterization of the data points are solved with iterative methods. Unlike direct surface fitting, there are surface reconstruction methods that are based on curve reconstruction, with which the surface-fitting problem is reduced to a curve-fitting problem. Park et al. [5] first reconstruct a set of cubic B-spline curves from cross-sectional data, after which surface is created by skinning these intermediate contour curves. Pottmann and Randrup [6] approximate rotational surfaces by applying the methods of line geometry to a set of surface normals. Benkő et al. [7] argue that extruded surfaces and revolved surfaces must be recognized and represented by means of swept profiles in reverse engineering. They first extract the extruding direction or revolving axis by optimizing an objective function based on the point cloud and/or its estimated differential properties. Then constrained fitting method is adopted to reconstruct consistent swept profiles composed of line and circular arc

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segments. If the swept surfaces are reconstructed by fitting a set of analytical surface instead, then the connectivity problems between these surfaces must be carefully dealt with. The connectivity is difficult, especially when dealing with smooth connections, because creating edge curves by surface-surface intersection are unstable in these situations. Although the connectivity problems can be solved by 3D constrained fitting (which will be mentioned later in this paper), solving the problem in 2D is easier and more stable. If the swept surfaces are reconstructed by free form surface fitting, the resulted surfaces cannot be modified by changing a few meaningful parameters to meet the general engineering needs. The same problem appears in Benkő's method for swept surface construction when the swept profiles are just reconstructed as freeform curves. Hence, 2D constrained fitting is crucial to reconstruct certain kinds of surfaces, i.e. extruded surfaces, revolved surfaces and lofted surfaces.

Constrained fitting is to fit data points with a set of geometric elements under constraints. Different from the conventional fitting techniques (in which data points is the only consideration), constrained fitting considers data points as well as geometric constraints, which makes the fitting process more robust to noise and hence generates more accurate geometric models. Moreover, geometric models reconstructed by constrained fitting are globally consistent. However, geometric models in conventional fitting are constructed locally and therefore not globally consistent. As a knowledge-based approach in the reverse engineering, constrained fitting has received attentions that it deserves. Porrill [8] introduces geometric constraints into reverse engineering to represent design intent for the first time. The constraints are used to assure the consistency of the model reconstruction by optimal combination of geometric elements. Porrill's work is focused on the construction of wire-frame model from sensor data. In this method, only lines and planes are covered. Werghi et al. [10] have done some valuable work on the problem of simultaneous constrained fitting of the multiple planar and quadric surfaces to range data. They made great progress in this area and gave a general framework for the integration of geometric constraints in reverse engineering. Sequential unconstrained minimization technique is used to convert the constrained optimization problem to unconstrained one. At each iteration, a new set of optimized parameters is obtained by minimizing the object function through using the standard Levenberg–Marquardt method. Recently, Benkő et al. [11] also did research on the issue of constrained fitting in reverse engineering. They proposed a new numerical method to solve the problem of conflicts among constraints. With this method, all constraints including inconsistent constraints are converted to the representation of independent constraints, which make the Lagrangian-multiplier method works more adequately for solving the optimization problem. In addition, the crucial processing time for each iteration is significantly reduced with efficient representation of objects and constraints, and appropriate approximations to distance functions.

In this paper, we focus on 2D constrained fitting, which is crucial to the reconstruction of extruded surfaces, revolved surfaces and lofted surfaces in engineering. Before discussing 2D constrained fitting, we assume that the data points have been sorted into a polyline, the segmentation has been done by either automatic methods or user interaction, and initial curves have been reconstructed by fitting the segmented data points already. Different from the 2D constrained fitting method in Benkő et al. [7,11], our 2D constrained fitting framework considers B-spline curves in addition to lines and circular arcs. In the numerical analysis, similar to Werghi et al. [10], we use a sequential unconstrained minimization technique to solve the constrained optimization problem. By using similarity transformation, we reduce the condition number of the Hessian matrix involved in the optimization process and therefore the numerical stability is significantly improved.

The rest of this paper is organized as follows. In Section 2, representations of geometric primitives, i.e. line, circle and B-spline curve, are presented, as well as the reasons for choosing such representations in 2D constrained fitting. In Section 3, we discuss the representations of geometric constraints among the geometric primitives. In Section 4, we describe the numerical stability associated with representations of geometric primitives, and provide a solution to this problem based on similarity transformation. In Section 5, we present several examples in real industrial applications to demonstrate the convergence and the stability of our method. Finally, conclusions are drawn in Section 6.

2. Representation of geometric primitives and objective functions

In our framework for 2D constrained fitting, line segments, circular arcs and B-spline curves are considered. Since constrained fitting problem are usually solved with iterative method, computational efficiency is crucial. Thus, profiles must be represented in the way such that objective functions can be easily evaluated.

2.1. Line

A line is represented by its analytic form with the usual implicit equation

$$f(x,y) = l_0 x + l_1 y + l_2 = 0,$$

where $\vec{X} = [l_0, l_1, l_2]$ is the parameter vector. Under normalization constraint $l_0^2 + l_0^2 - 1 = 0$, $f(x,y)$ is exactly the geometric distance between point (x,y) and the line (regardless of the sign). The sum of the squared distances