Applied Soft Computing 13 (2013) 1792–1799

Contents lists available at SciVerse ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Fuzzy analytic hierarchy process and analytic network process: An integrated fuzzy logarithmic preference programming

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ARTICLE INFO

Article history:
Received 2 May 2011
Received in revised form 16 August 2012
Accepted 24 November 2012
Available online 4 January 2013

Keywords:
Analytic network process
Analytic hierarchy process
Inconsistency
Fuzzy preference programming
Multiplicative constraints

ABSTRACT

This paper proposes a two-stage fuzzy logarithmic preference programming with multi-criteria decision-making, in order to derive the priorities of comparison matrices in the analytic hierarchy pprocess (AHP) and the analytic network process (ANP). The Fuzzy Preference Programming (FPP) proposed by Mikhailov and Singh [L. Mikhailov, M.G. Singh, Fuzzy assessment of priorities with application to competitive bidding, Journal of Decision Systems 8 (1999) 11–28] is suitable for deriving weights in interval or fuzzy comparison matrices, especially those displaying inconsistencies. However, the weakness of the FPP is that it obtains priorities of comparison matrices by additive constraints, and generates different priorities by processing upper and lower triangular judgments. In addition, the FPP solves the comparison matrix individually. By using multiplicative constraints, the method proposed in this paper can generate the same priorities from upper and lower triangular judgments with crisp, interval or fuzzy values. Our proposed method can solve all of the matrices simultaneously by multiple objective programming. Finally, five examples are demonstrated to show the proposed method in more detail.

1. Introduction

The analytic hierarchy process (AHP) has been widely used in multi-criteria decision-making to measure tangible and intangible criteria [2]. AHP aims to decompose the decision-making process into a hierarchical structure, assuming that the relationships of criteria in different levels are independent. Since its development, many applications have been published, such as portfolio selection [3], facility location selection [4], reverse logistics [5], etc.

Conventional AHP only uses crisp pair-wise judgments to derive weights without considering the uncertainty of human intuition. Thus, Satty and Vargas [6] proposed an interval AHP to deal with interval judgments. Unfortunately, when using this approach for interval judgments, the measurement of inconsistencies, while generating weights, becomes difficult.

Subsequently, Islam et al. [3] proposed Lexicographic Goal Programming (LGP) to handle the inconsistency problems in the interval AHP by using deviation variables. Mikhailov and Singh [1] proposed the fuzzy preference programming (FPP) method, which derived crisp priorities from interval or fuzzy comparison matrices by introducing tolerance parameters. However, due to the additive constraints for generating weights in the FPP, different priorities and rankings from the upper and lower triangular judgments could be obtained [7,8]. Wang [9] proposed that the LGP has the same problem with different priorities and rankings from upper and lower triangular judgments. In fact, the upper and lower triangular judgments of a comparison matrix provides the same information on the preference of weights, but generates different weights. Therefore, the question arises as to which weight vector and ranking is more accurate [7–9].

A two-stage logarithmic goal-programming method ensures that the same interval weights are generated from both upper and lower triangular judgments of an interval comparison matrix [9]. Chandran et al. [10] applied a two-stage linear programming (LP) model to determine weights, using logarithms for crisp or interval inconsistent matrices. Their approaches adopted the geometric mean for interval judgments. Unfortunately, it had too many constraints with regard to the number of weights.

The performing time or complexity of the aforementioned methods is proportional to the number of comparison matrices. If more interval weights need to be obtained, a longer processing time is required, which leads to complexity. As a result, multiple objective programming has been proposed to obtain all priorities simultaneously [11].

In a real-life environment, many problems exist with interdependence relations. These are difficult to present by means of a hierarchical structure. To cope with more practical real-life problems, the analytic network process (ANP) was proposed by Satty [12]. The ANP is capable of addressing interdependent

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http://dx.doi.org/10.1016/j.asoc.2012.11.015

relationships among criteria and sub-criteria [13]. In fact, the ANP is a generalization of the AHP, and extends the AHP to the problems with dependence and feedback among criteria by using a network structure and supermatrix approach. The crisp ANP approach has been applied to such processes as economic forecasting [14] and successful system factor selection [15]. Moreover, both AHP and ANP are effective in regard to prediction [16].

To handle uncertain judgments with inconsistency in the ANP, Mikhailov and Singh [17], and Mikhailov [18] applied the FPP in fuzzy and interval ANP. The advantage of the FPP is that it obtains crisp priorities from interval and fuzzy judgments straightforwardly within the ANP for decision-making under uncertain conditions, while all the other known interval and fuzzy prioritization methods, which derive interval or fuzzy priorities, cannot be used directly in the ANP.

The initial weight-generating process of the ANP is the same as that of the AHP. Each comparison matrix must be constructed as an individual FPP model. When the network structure is more complex, more complexity is required to perform the FPP models [11]. Because of additive constraints in the FPP [7,8], the issue of different weights obtained from the lower and upper triangular in the ANP or fuzzy ANP remains.

Thus, a two-stage fuzzy logarithmic preference programming with multiple objective programming is proposed to simultaneously derive priorities in fuzzy AHP or ANP. Our proposed method can handle complex problem structure, regardless of the number of comparison matrices, because all of the priorities are generated simultaneously. Moreover, the proposed method can generate the same priorities from upper and lower triangular judgments by multiplicative constraints [19] instead of additive constraints, as demonstrated in the five examples.

The remainder of this paper is organized as follows. Section 2 describes the FFP method in detail; Section 3 presents the proposed method; Section 4 discusses comparative results of five examples; Section 5 offers conclusions.

2. The FPP model

The FPP deals with fuzzy comparison matrices, particularly handling inconsistent fuzzy matrices [17]. Let $\tilde{A} = \{a_{ij}\}$ represent the fuzzy pair-wise comparison matrix with n criteria.

$$\tilde{A} = \begin{bmatrix} 1 & \tilde{a}_{12} & \tilde{a}_{13} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & \tilde{a}_{23} & \cdots & \tilde{a}_{2n} \\ \tilde{a}_{31} & \tilde{a}_{32} & 1 & \cdots & \tilde{a}_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \tilde{a}_{n3} & \cdots & 1 \end{bmatrix}$$

where $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$, $\tilde{a}_{ji} = 1/\tilde{a}_{ij} = (1/u_{ij}, 1/m_{ij}, 1/l_{ij})$, l_{ij}, m_{ij} and u_{ij} are the lower, center and upper bounds of the corresponding uncertainty judgments for $i = 1, 2, \ldots, n-1$, $j = 2, 3, \ldots, n$, $j > i$. The definitions of the basic arithmetic operations on fuzzy numbers are taken from Dubois and Prade [20].

When the interval judgments are consistent, the priority vector $w = (w_1, w_2, \ldots, w_n)^T$, whose elements satisfy the inequalities:

$$l_{ij}(\alpha) \le \frac{w_i}{w_j} \le u_{ij}(\alpha), \quad i = 1, 2, \ldots, n-1, \quad j = 2, 3, \ldots, n, \quad j > i, \tag{1}$$

where α-cut is ranging from $0 \le \alpha \le 1$.

$$l_{ij}(\alpha) = l_{ij} + \alpha(m_{ij} - l_{ij}), \tag{2}$$

$$u_{ij}(\alpha) = u_{ij} + \alpha(m_{ij} - u_{ij}). \tag{3}$$

When the judgments are inconsistent, a priority vector that satisfies all fuzzy judgments approximately is presented as below:

$$l_{ij}(\alpha) \tilde{\le} \frac{w_i}{w_j} \tilde{\le} u_{ij}(\alpha), \quad i = 1, 2, \ldots, n-1, \quad j = 2, 3, \ldots, n, \quad j > i, \tag{4}$$

where $\tilde{\le}$ denotes the statement "fuzzy or equal to". Eq. (4) can be represented as a set of single-side fuzzy constraints:

$$\begin{cases} w_i - w_j u_{ij}(\alpha) \tilde{\le} 0, \\ -w_i + w_j l_{ij}(\alpha) \tilde{\le} 0. \end{cases} \tag{5}$$

The above set of $n(n-1)$ fuzzy constraints can be given in a matrix form as following:

$$\boldsymbol{Rw} \tilde{\le} 0, \tag{6}$$

where the matrix

$$\boldsymbol{R} \in \Re^{m \times n}, \ m = n(n-1). \tag{7}$$

To measure the consistent satisfaction of the fuzzy judgment, the linear membership function is adopted as follows:

$$\mu_k(R_k w) = \begin{cases} 1 - \dfrac{R_k w}{d_k} & R_k w \le d_k, \\ 0 & R_k w \ge d_k, \end{cases} \tag{8}$$

where d_k is a tolerance parameter for kth constraint, given by the decision maker for $k = 1, 2, \ldots, m$. In general, d_k is set as 1.

The FPP method selects a priority vector with the highest degree of membership as described below:

$$\lambda = \max_{w} \left[\min\{\mu_1(R_1 w), \mu_2(R_2 w), \ldots, \mu_m(R_n w)\} | \sum_{i=1}^{n} w_i = 1 \right]. \tag{9}$$

The solution of the prioritization problem at each α-cut level can be obtained by solving the linear programming problem as follows:

Max λ

s.t.

$$d_k \lambda + w_i - u_{ij}(\alpha) w_j \le d_k, \tag{10}$$

$$d_k \lambda - w_i + l_{ij}(\alpha) w_j \le d_k, \tag{11}$$

$$\sum_{i=1}^{n} w_i = 1, \quad w_i > 0, \tag{12}$$

$$0 \le \lambda \le 1, \quad i = 1, 2, \ldots, n-1, \quad j = 2, 3 \ldots, n, j > i, \quad k = 1, 2, \ldots, m.$$

When the value of λ is equal to 1, the comparison matrix is consistent where the value of λ is less than 1, the comparison matrix is inconsistent.

The above linear program is the core of the FPP method used for deriving the priorities of fuzzy AHP [18]. Moreover, the FPP can be applied in ANP, which generalizes AHP without making assumptions about the dependence of higher level elements from lower elements in a hierarchy, or about the independence of elements within the same levels [16,21]. The following are the eight main steps to apply the FPP in the ANP case [11,17]:

Step 1: Construct a hierarchical or network structure relative to the decision goal.

Step 2: Identify the dependences among all components of the above structure to define the impact between each.

Step 3: Fill in pair-wise comparison matrices of the criteria with crisp, interval or fuzzy judgments.