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Self-efficacy and short-term memory capacity as predictors of proportional reasoning

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ABSTRACT

This study investigated the influence of self-efficacy and working memory on time-pressured completion of a test of proportional reasoning. Participants completed standard working memory measures of storage as well as storage and transformation (verbal and numerical), reported their self-efficacy with numerical information and attempted a time-restricted numeracy task. The task was composed of problems featuring fractions, percentages and probabilities, which included simple problems requiring mainly memory retrieval, as well as problems that required calculation and operation-switching. While short-term memory and self-efficacy played a role in performance, self-efficacy explained a significant proportion of the variance in performance above and beyond the effects of short-term memory. As similar findings have recently been shown for mental multiplication, the current study suggests that this relationship between working memory capacity and self-efficacy also holds for proportional reasoning. Correlational analyses found gender differences in the relation between self-efficacy and proportional reasoning.

Competency with numbers is a complex skill relying on various interwoven abilities (Mazzocco, 2008). While task-experience is an important factor in solving mathematical problems accurately, approximately half of the variance in mathematics performance can be attributed to factors other than ability (Suinn & Edwards, 1982). These factors include self-efficacy beliefs (Hoffman, 2010; Hoffman & Schraw, 2009; Lopez, Lent, Brown, & Gore, 1997; Pajares & Kranzler, 1995; Pajares & Miller, 1994; Williams & Williams, 2010) and working memory capacity (DeStefano & LeFevre, 2004; Jackson & Warrington, 1986; Klein & Bisanz, 2000; Seitz & Schumann-Hengsteler, 2000, 2002; Swanson, 2004; Swanson & Beebe-Frankenberger, 2004).

Self-efficacy can be defined as a person's belief in their own ability to carry out a certain task successfully (Bandura, 1986). The positive role of self-efficacy in mathematics performance is apparent even when factors such as mathematics ability (Pajares & Kranzler, 1995; Pajares & Miller, 1994), gender (Pajares & Miller, 1994) and prior knowledge (Pajares & Kranzler, 1995) are controlled. More recently, self-efficacy has also been found to explain significant variance in arithmetic performance beyond the effects of working memory capacity (Hoffman, 2010; Hoffman & Schraw, 2009). However, as these recent studies investigated mental arithmetic (multiplication), it is not clear if the same findings would hold for other numerical tasks. Hoffman and Schraw (2009) found that self-efficacy was a significant predictor of performance on a mental multiplication task when controlling for other variables, consistent with the motivational efficiency hypothesis (Hoffman & Spatariu, 2008), which predicts that motivational factors play a key role in mathematical problem solving. Accordingly, self-efficacy boosts performance through focusing attention and strategy use in situations with increased working memory demands or time constraints (Hoffman & Schraw, 2009). However, Hoffman and Schraw note that their multiplication task involves automatic memory retrieval and more complex problem solving tasks might produce a different pattern of results. Their suggestion that self-efficacy's effect on performance be examined using problems that might place greater demand on working memory is the basis for the present study.

Unlike mental arithmetic, which is often a well-practiced task, numeracy reflects proficiency with basic numerical concepts and their application in real-world scenarios (e.g. Lipkus, Samsa, & Rimer, 2001). This encompasses a broad set of skills including proficiency with basic mathematical functions (e.g. calculation, fractions, algebra and geometry), as well as applying these skills to concepts of time, money, measurement, probability and health risk perception (Rothman, Montori, Cherrington, & Pignone, 2008). In a given scenario, proficient numeracy also involves ascertaining which numerical skills to apply in order to solve a given problem (Montori & Rothman, 2005). Numerate individuals thus know which mathematical skills would be most efficient for problem solving in the absence of explicit instruction. Earlier measures of numeracy were used mainly to describe individuals' mathematics ability for educational purposes; however, research in the last decade has reconstructed the concept of numeracy to reflect an individual's broader, more general understanding of number (Brooks & Pui, 2010). From a practical perspective, numeracy has been widely studied in relation to medical risk communication and perception (see Reyna & Brainerd, 2007 for a review), which mostly focuses on proportional reasoning (e.g. fractions, percentages and probability; e.g. Lipkus et al., 2001; Peters et al., 2006; Rothman et al., 2008). These studies

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highlighted concerns over low numeracy (e.g. Jukes & Gilchrist, 2006; Lipkus et al., 2001; Schwartz, Woloshin, Black, & Welch, 1997). Lipkus et al. (2001), for example, found that 16–20% of well-educated adults failed to answer the simplest probability questions correctly (e.g. which presents the greatest risk of getting a disease, 1%, 5% or 10%) and also noted errors when participants were required to translate between metrics (e.g. from percentages to fractions). While these findings are somewhat surprising, everyday applications of numerical knowledge are likely to require rapid switching between arithmetic operations and formats without explicit instruction, as well as a firm grasp of core numerical concepts, such as probability, and efficient memory retrieval. These skills might thus differ from those required in arithmetic tasks that isolate a specific operation (e.g., multiplication tasks).

The role of self-efficacy in mathematical problem solving has been greatly emphasized (e.g. Hoffman, 2010; Hoffman & Schraw, 2009; Pajares & Kranzler, 1995; Pajares & Miller, 1994). However, considerably less research has focused on its role in proportional reasoning specifically and little data exist regarding proportional reasoning in adults. In an attempt to measure numeracy without an objective test, which is often anxiety-inducing for participants, Fagerlin et al. (2007) developed the Subjective Numeracy Scale (SNS), an eight-item measure of self-efficacy with proportional reasoning (percentages and fractions) and preference for information in numerical (e.g. "a 1% chance") or verbal (e.g. "a rare chance") format. The SNS correlated highly with the objective numeracy measure of Lipkus et al. (2001), which features problems that involve fractions, percentages, ratios and probabilities. The SNS was also subsequently validated as having significant predictive ability as regards participants' medical risk comprehension and decision making capabilities (e.g. hypothetical scenarios and outcome probabilities were presented and participants had to choose which treatment would be most effective over time; see Zikmund-Fisher, Smith, Ubel, & Fagerlin, 2007). McMullan, Jones, and Lea (2012) have also recently argued the case for considering motivational factors such as mathematics anxiety and self-efficacy in drug calculation ability of nursing students, which relies to a great extent on proportional reasoning skill (e.g. calculating dosage ratios, percentage solutions, infusion rates and drip rates). Self-efficacy played a significant role in drug calculation ability and was also found to mediate the effects of mathematics anxiety, namely that increased self-efficacy was associated with lower mathematics anxiety and vice versa. These studies suggest a positive role for self-efficacy in numeracy; however, more focus is needed on the role of self-efficacy in proportional reasoning per se and how it might mediate the effects of other variables that have been shown to play a role in mathematical problem solving, such as working memory.

With regard to the relationship between working memory and mathematics, studies often investigate specific problem-solving situations such as single and multi-digit arithmetic. Such studies generally do not consider everyday numeracy problems that require reasoning with fractions, percentages, and probabilities. Although the positive influence of high working memory capacity on mathematical problem solving is well established (e.g. Adams & Hitch, 1997; DeStefano & LeFevre, 2004; Jackson & Warrington, 1986; Klein & Bisanz, 2000; Swanson & Beebe-Frankenberger, 2004), it seems to depend on the specific problem-solving situation (DeStefano & LeFevre, 2004) as well as the measures of working memory employed (see Raghubar, Barnes, & Hecht, 2010, for review). Different components of the working memory system are activated depending on operation, strategy use and individual differences (Raghubar et al., 2010). In single-digit addition and subtraction, for example, central executive function usually comes into play across operations, whereas the phonological loop only plays a role where counting strategies are required (Hecht, 2002; Imbo & Vandierendonck, 2007; Seyler, Kirk, & Ashcraft, 2003). Phonological involvement is generally not found for single-digit multiplication, which largely depends on direct long-term memory retrieval (De Rammelaere, Stuyven, & Vandierendonck, 2001; Seitz & Schumann-Hengsteler, 2000, 2002). These studies thus illustrate the involvement of different working memory components in specific problem-solving contexts. However, they do not generalize to everyday numeracy, which involves proportional reasoning and many different problem-solving steps, which are not explicitly instructed.

The influence of working memory on children's proportional reasoning has been explored to some extent (e.g. Berg, 2008; Hecht, Close, & Santisi, 2003). Hecht et al. (2003), for example found that working memory (as measured by counting span) played a significant role in children's comprehension of word problems involving fractions, beyond effects of prior mathematical knowledge. Considering the concerns over low numeracy of adult samples, more research that considers the role of working memory in the proportional reasoning skills of adults is warranted.

In a recent review, Raghubar et al. (2010) note that there are still few studies that consider factors that might influence the complex relationship between working memory and mathematics. The recent studies of Hoffman and Schraw (2009) and Hoffman (2010) were important in that they considered motivational factors such as self-efficacy, and they found that self-efficacy continued to play an important role in performance in a multiplication task when demands were placed on working memory capacity (see Hoffman & Schraw, 2009). The proposed motivational efficiency hypothesis (see Hoffman & Schraw, 2009) suggests that self-efficacy plays a key role beyond the influence of working memory capacity not just in problem solving accuracy, but also in problem solving efficiency (the ratio of problems correctly solved to the time needed to solve them).

The current study aimed to extend these findings to everyday numeracy that involves proportional reasoning. To measure problem-solving performance we used a time-restricted proportional reasoning test with items ranging from simple to more complex problems, based on the numeracy test developed by Lipkus et al. (2001). We measured participants' self-efficacy for numerical problem solving with the ability sub-scale of the Subjective Numeracy Scale developed by Fagerlin et al. (2007). To measure working memory, we used standard verbal and numerical span tasks differing in complexity in order to measure both the storage and processing components of working memory, namely forward digit span, backward digit span and sentence span. Forward digit span typically measures short-term memory storage capacity (e.g. Daneman & Carpenter, 1980; Oberauer, 1993), whereas backward digit span and sentence span measure storage as well as processing components of working memory (e.g. Kyllonen & Christal, 1990; Oberauer et al., 2000; Raghubar et al., 2010). During the backward digit span task, participants hear a sequence of digits and are required to repeat the digits back verbally in reversed order. Participants thus have to store the heard sequence of digits in short-term memory and also transform them mentally in order to repeat them in reversed order, which involves the phonological loop and central executive function (e.g. Raghubar et al., 2010). The forward digit span task, on the other hand, requires only the storage function of short-term memory (phonological) and does not require the processing component, which comes into play during the backward digit span task. The sentence span task offers a greater level of complexity and involves answering true/false to trivial sentences (e.g. "Football is a sport") while also trying to remember the last word of each sentence in a sequence of sentences. Similar to the digit backward span task, this task thus involves simultaneous storage and transformation and is also likely to require central executive function in order to initiate/inhibit the true/false responses to the sentences (Oberauer et al., 2000). The working memory tasks thus included two storage and transformation measures of working memory (numerical and verbal) and one measure of short-term memory. Hoffman and Schraw (2009) found that working memory as measured on a complex task (storage and transformation functions) did not play a strong and consistent role in problem-solving efficiency in arithmetic. Consistent with the motivational efficiency hypothesis (Hoffman & Spatariu, 2008), self-efficacy played a significant role in problem-solving efficiency at different levels of problem complexity, above and beyond the effects of working memory. Based on these findings in relation to mental arithmetic