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A granulation of linguistic information in AHP decision-making problems

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ABSTRACT

To be fully utilized, linguistic information present in decision-making, has to be made operational through information granulation. This study is concerned with information granulation present in the problems of Analytic Hierarchy Process (AHP), which is available in the characterization of a pairwise assessment of alternatives studied in the decision-making problem. The granulation of entries of reciprocal matrices forming the cornerstone of the AHP is formulated as a optimization problem in which an inconsistency index is minimized by a suitable mapping of the linguistic terms on the predetermined scale. Particle Swarm Optimization is used as an optimization framework. Both individual and group decision-making models of AHP are discussed.

1. Introduction

In decision-making problems, we are commonly faced with information provided by humans, which is inherently non-numeric. Partial evaluations, preferences, weights are expressed linguistically. The evident role of fuzzy sets in decision-making and associated important processes such as e.g., consensus building is well documented in the literature, see [1,5,6,12].

While fuzzy sets have raised awareness about the non-numeric nature of information, its importance, a need for its handling and provided a great deal of techniques of processing fuzzy sets, the fundamental issue about a transformation of available pieces of linguistic information into formal constructs of information granules. The resulting information granules are afterwards effectively processed within the computing setting pertinent to the assumed framework of information granulation. The linguistic terms such as high, and medium are in common usage. It is not clear, however, how they have to be translated into the entities, which can be further seamlessly processed using the formalisms of sets, fuzzy sets, rough sets and alike. Likewise, it is not straightforward what optimization criterion can be envisioned when arriving at the formalization of the linguistic terms through information granules.

Given the diversity of decision-making problems and being alerted to the fact that each of their categories could come with some underlying specificity and particular requirements, in this study we concentrate on the Analytic Hierarchy Process (AHP) model, which addresses a large and important category of decision-making schemes, see [13,15,16]. There is a visible abundance literature on the refinements and generalizations of these models as well as various applied studies [2,3,7,8,14].

The pairwise comparisons of alternatives are articulated in terms of linguistic quantifications, say *highly* preferred, *moderately* preferred, etc. Each term is associated with some numeric values. It has been identified quite early in the development of the AHP-like models that the single numeric values taken from the 1–9 scale do not necessarily fully reflect the complexity of the non-numeric nature of the pairwise comparisons. The first approach along this line was presented in [9] where the authors admitted triangular fuzzy numbers defined in the scale used in the method, say 1–9. There have been a significant number of pursuits along this line. The granular nature of the pairwise assessments was discussed in the context of a group decision-making where reaching consensus calls for some flexibility of evaluations individual assessments have to be endowed with to facilitate processes of consensus building.

In the study, we focus on two scenarios: decision-making involving a single decision – maker and decision-making realized in presence of several decision-makers (group decision-making problem). The granulation formalism being discussed in the study concerns intervals and fuzzy sets however it applies equally well to any other formal scheme of information granulation, say probabilistic sets, shadowed sets, rough sets. It is worth stressing here that information granulation offers an *operational* model of the AHP to

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be used in presence of linguistic pairwise comparisons. The PSO framework supporting the formation of information granules helps translate linguistic quantification into meaningful information granules so that the highest consistency of the evaluations is achieved.

The presentation is structured as follows. We start with a brief outline of the AHP decision model (Section 2). Section 3 is concerned with a quantification of linguistic terms present in reciprocal matrices where a granulation of the terms leads to the operational realization of further processing forming preference vectors. The optimization of the granulation process is elaborated on in Section 4. Both set-based and fuzzy set-oriented information granulation mechanisms are studied. Group decision-making in which a collection of reciprocal matrices is considered is covered in Section 5. Conclusions are drawn Section 6. The study comes with a series of numeric examples, which in a step-by-step manner offer an illustration of the presented ideas and the ensuing algorithms.

2. The AHP method – a brief overview

Given a finite number of alternatives, say various options, solutions, etc. $a_1, a_2, \ldots, a_n$, etc. to be considered in a certain investment scenario, the objective is to order them by associating with them some degrees of preference expressed in the [0, 1] interval. The essence of the method introduced by Saaty is to determine such preference values through running a series of pairwise comparisons of the alternatives. The results are organized in an $n \times n$ reciprocal matrix $R = [r_{ij}]$, $i, j = 1, 2, \ldots, n$. The matrix exhibits two important features. The diagonal values of the matrix are equal to 1. The entries that are symmetrically positioned with respect to the diagonal satisfy the condition of reciprocality that is $r_{ij} = 1/r_{ji}$.

The starting point of the estimation process of the fuzzy set of preferences are entries of the reciprocal matrix which are obtained through collecting results of pairwise evaluations offered by an expert, designer or user (depending on the character of the task at hand). Prior to making any assessment, the expert is provided with a finite scale with values in-between 1–7. Some other alternatives of the scales such as those involving 5 or 9 levels could be sought as well. If a_i is strongly preferred over a_j when being considered in the context of the fuzzy set whose membership function we would like to estimate, then this judgment is expressed by assigning high values of the available scale, say 6 or 7. If we still sense that a_i is preferred over a_j yet the strength of this preference is lower in comparison with the previous case, then this is quantified using some intermediate values of the scale, say 3 or 4. If no difference is considered, the values close to 1 are the preferred choice, say 2 or 1. The value of 1 indicates that a_i and a_j are equally preferred. The general quantification of preferences positioned on the scale of 1–9 can be described as follows:

equal importance 1,
moderate importance of one element over another 3,
strong importance 5,
demonstrated importance 7,
extreme importance 9.

There are also some intermediate values, which could be used to further quantify the relative dominance. On the other hand, if a_j is preferred over a_i, the corresponding entry assumes values below one. Given the reciprocal nature of the assessment, once the preference of a_i over a_j has been quantified, the inverse of this number is inserted into the entry of the matrix that is located at the (j, i)-th coordinate. Next the maximal eigenvalue is computed along with its corresponding eigenvector. The normalized version of the eigenvector is then the membership function of the fuzzy set we considered when doing all pairwise assessments of the elements of its universe of discourse. The effort to complete pairwise evaluations is far more manageable in comparison to any experimental overhead we need when assigning membership grades to all elements (alternatives) of the universe in a single step. Practically, the pairwise comparison helps the expert focus only on two elements once at a time thus reducing uncertainty and hesitation while leading to the higher level of consistency. The assessments are not free of bias and could exhibit some inconsistency. In particular, we cannot expect that the transitivity requirement could be fully satisfied. Fortunately, the lack of consistency could be quantified and monitored. The largest eigenvalue computed for R is always greater than the dimensionality of the reciprocal matrix (recall that in reciprocal matrices the elements positioned symmetrically along the main diagonal are inverse of each other), $\lambda_{max} > n$ where the equality $\lambda_{max} = n$ occurs only if the results are fully consistent. The ratio

$$v = (\lambda_{\max} - n)/(n - 1) \quad (1)$$

can be regarded as a certain consistency index of the data; the higher its value, the less consistent are the collected experimental results. This expression can be sought as the indicator of the quality of the pairwise assessments provided by the expert. If the value of v is too high exceeding a certain superimposed threshold, the experiment may need to be repeated. Typically if v is less than a certain threshold level, the assessment is sought to be consistent while higher values of v call for the re-examination of the experimental data and a re-run of the experiment. The threshold of the consistency ratio (CR) expressed as the ratio of the consistency index, CI and the random consistency index (RI), CR = CI/RI is also established (typically assuming the value of 0.1) to assess the quality of the results of pairwise comparison, cf. [4].

3. A quantification (granulation) of linguistic terms as their operational realization

The linguistic terms used in a pairwise comparison of alternatives are expressed linguistically by admitting qualitative terms. They can be organized in a linear fashion, as there is some apparent linear order among them. The terms themselves are not operational meaning that no further processing can be realized, which involves a quantification of the linguistic terms. Schematically, we can portray the process of arriving at the operational representation of linguistic terms as illustrated in Fig. 1. In this figure, capital letters denote the corresponding linguistic terms: L – *low*, M – *medium*, H – *high*.

The two important features of such granulation mechanisms are worth noting here: (a) the mapping is by no means linear that is a localization of the associated information granules on the scale is not uniform, (b) the semantics of the terms allocated in the process of granulation is retained. Various information granulation formal-

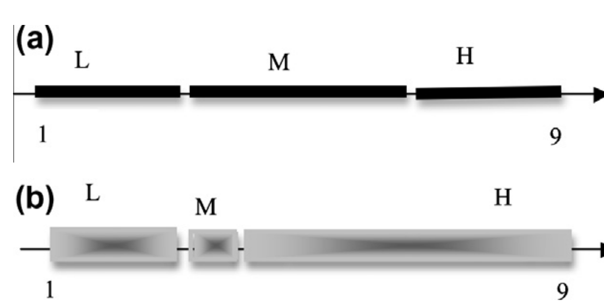

Fig. 1. Towards the operational realization of linguistic terms: (a) realization with the aid of intervals, and (b) fuzzy set-based implementation.

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