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Algebra in a man with severe aphasia

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Abstract

We report a dissociation between higher order mathematical ability and language in the case of a man (SO) with severe aphasia. Despite severely impaired abilities in the language domain and difficulties with processing both phonological and orthographic number words, he was able to judge the equivalence of and to transform and simplify mathematical expressions in algebraic notation. SO was sensitive to structure-dependent properties of algebraic expressions and displayed considerable capacity to retrieve algebraic facts, rules and principles, and to apply them to novel problems. He demonstrated similar capacity in solving expressions containing either solely numeric or abstract algebraic symbols (e.g., $8-(3-5)+3$ versus $b-(a-c)+a$). The results show the retention of elementary algebra despite severe aphasia and provide evidence for the preservation of symbolic capacity in one modality and hence against the notion of aphasia as asymbolia.

Keywords: Language; Number; Mathematics; Aphasia; Algebra; Asymbolia

1. Introduction

The role of language in mathematics is a hotly debated issue in the cognitive sciences (Brannon, 2005; Gelman & Butterworth, 2005; Gelman & Gallistel, 2004) and several lines of enquiry contribute to the debate. Studies with animals and preverbal human infants have demonstrated basic numerical and calculation abilities (Dehaene, Dehaene-Lambertz, & Cohen, 1998; Flombaum, Junge, & Hauser, 2005; Wynn, 1992), indicating the autonomy of basic number capacity from language. However, recent cross-linguistic studies demonstrated language effects on the development of basic calculation skills (Houde & Tzourio-Mazoyer, 2003) and on number representation and comparison (Nuerk, Weger, & Willmes, 2005). Studies investigating numerical abilities in Amazonian tribes, who possess only a rudimentary lexicon for number words, point to possible limitations for the acquisition of mathematics without supporting linguistic lexical resources (Gordon, 2004; Pica, Lemer, Izard, & Dehaene, 2004). Similarly, experiments with bilingual adults have shown a language training effect when performing exact as opposed to approximate calculations (Spelke & Tsivkin, 2001).

Further evidence for a possible role of language in calculation comes from functional imaging studies. Some studies have found evidence for the recruitment of left hemisphere perisylvian language areas during exact calculation (Cohen, Dehaene, Chochon, Lehericy, & Naccache, 2000; Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999). However, others have implicated the involvement of a bilateral parietofrontal network and the bilateral inferior temporal gyri, including areas associated with visuospatial working memory and mental imagery, during mental arithmetic (Houde & Tzourio-Mazoyer, 2003; Venkatraman, Ansari, & Chee, 2005; Zago et al., 2001).

In the field of neuropsychology, although aphasia is often associated with impaired number and calculation ability (Delazer & Bartha, 2001; Delazer, Girelli, Semenza, & Denes, 1999), dissociations between language and mathematics have also been demonstrated. Double dissociations between aphasia and acalculia have been described with reports of preserved language skills despite impaired mathematical abilities (Butterworth, 1999), or retained mathematical skills despite severely impaired language (Rossor, Warrington, & Cipolotti, 1995; Varley, Klessinger, Romanowski, & Siegal, 2005). These studies indicate considerable autonomy in the neurocognitive mechanisms underpinning language and calculation.

Compared to the rich body of evidence on number processing and elementary arithmetic, relatively little is known about the cognitive processes involved in more complex and advanced

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mathematical abilities such as solving fractions or algebraic expressions (Butterworth, 1999; Dehaene, 1997). Similarly, the role language plays in these processes is still to be determined (Campbell & Epp, 2004). There are few neuropsychological studies which report on higher mathematical functions in brain damaged patients.

With regard to algebra, one study reported the case of DA, who despite severe disruptions in basic calculation abilities could still solve abstract algebraic equations (Hittmair-Delazer, Sailer, & Benke, 1995). DA was impaired in solving simple addition, subtraction, division or multiplication problems, but could correctly simplify abstract expressions such as $(b \times a) \div (a \times b)$ or $(a+b)+(b+a)$ and make correct judgements whether abstract algebraic equations like $b-a=a-b$ or $(d \div c)+a=(d+a) \div (c+a)$ were true or false. In the study, algebraic expressions were used to test for conceptual knowledge of arithmetic. Conceptual knowledge entails deeper understanding of arithmetical operations and laws pertaining to these operations (Semenza, 2002), such as commutativity for addition (i.e., $4+7=7+4$), or non-commutativity for subtraction (i.e., $7-4 \neq 4-7$). Similarly, transforming and judging equivalences of expressions in algebra requires knowledge of such mathematical principles (Resnick, Cauzinille-Marmeche, & Mathieu, 1987). DA showed a dissociation between impaired arithmetical fact knowledge (such as $2+3$) and intact knowledge of arithmetical procedures (such as sequences of necessary steps to perform multi-digit calculations) and conceptual knowledge (e.g., $a+b=b+a$). The findings from DA have been interpreted as evidence for two parallel, independent levels of mathematical processing, one formal-algebraic and one arithmetical–numerical (Hittmair-Delazer et al., 1995), and further, as preliminary evidence for the existence of independent neuronal circuits for algebraic knowledge and mental calculation (Dehaene, 1997).

Despite the claim of autonomy between algebraic and calculation processes, some functional imaging studies reveal common activations across these behaviours. Anderson, Qin, Sohn, Stenger, and Carter (2003) found that brain regions active during algebra equation solving in adults were also areas involved in calculation and mental arithmetic (e.g., Dehaene, Piazza, Pinel, & Cohen, 2003; Dehaene et al., 1999; Gruber, Indefrey, Steinmetz, & Kleinschmidt, 2001; Menon, Rivera, White, Glover, & Reiss, 2000; Zago et al., 2001). These included the left intraparietal sulcus, left precuneus, left inferior frontal gyrus, left pre-frontal regions and left and right supramarginal gyri.

However, the question regarding the relation of language and elementary algebra has not been directly addressed in a neuropsychological case study to date. As DA's language abilities were intact, his case did not show whether algebraic reasoning can be sustained in the face of severe language impairment. With regard to the relation between natural language and algebra, there are similarities between the two systems. In algebra, there is a 'syntactic' level of mathematical processing which is necessary to determine whether an algebraic equation is well or ill-formed (e.g., $a \div (b-(c+a))$ versus $a\ (\div b-(c+a))$, similar to syntactic processing in natural language (e.g., *the man hits the ball* versus *the man the ball hits*). Similarly, an algebraic expression contains operator signs, abstract variables (denoted as letters, e.g., a or b) and numerals (such as 3 or 5 expressing specific quantities) which constitute a symbolic system and are analogous to the language lexicon (with comparable abstract and concrete words, e.g., *creature* versus *dog*). However, it has still to be determined whether syntactic and symbolic capacities in algebra are related to natural language or whether they are largely independent.

We address this question in the current investigation, and report the case of a man (SO) with severe aphasia who retained a capacity for solving algebraic expressions. SO also retained calculation ability and his performance in number processing and basic calculation tests has been reported elsewhere (Varley et al., 2005). In summary, SO displayed good understanding and production of Arabic numerals, but had mild impairment in processing phonological number words, and marked disruption of processing orthographic number words. He was able to transcode numbers across different formats, with intact performance in matching Arabic numerals to orthographic number words, and only mild impairment in other forms of transcoding. SO was accurate on an analogue estimation task, and was able to perform all basic single-digit arithmetical operations, and addition and subtraction of two-digit operands. He had some difficulty in multiplication and division of two-digit multipliers and divisors. He was able to add and subtract fractions and was able to calculate the result of mathematical expressions containing brackets. SO was the only participant in the previously reported case series who pre-morbidly had a high level competence in mathematics. Thus, with regard to arithmetic, SO demonstrated largely intact knowledge of arithmetical facts and procedures and intact conceptual knowledge in face of severe aphasia.

In this report, we examine whether higher order mathematics in the form of elementary algebra was retained despite severe language impairment and whether language is necessary to gain access to the algebraic-formal level of mathematical processing. In particular, SO's capacity to comprehend and solve mathematical expressions including terms with not only numbers but also abstract variables (represented as letters, e.g., a, b or y) was explored.

2. Method

2.1. Participants

SO and five healthy male volunteers participated in the study. All participants gave informed consent prior to participation in the study, and the protocol was approved by the North Sheffield Research Ethics Committee (NS200291449). SO was a 56-year-old retired university professor. He had been a professor in Faculty of Science and, although he was not a professor of mathematics, the research in which he was engaged required an advanced competence in the subject. He was pre-morbidly right-handed and, at the time of the investigation, was 3.5 years post-onset of a left hemisphere vascular lesion. A CT scan revealed a large lesion resulting from a proximal occlusion of the left middle cerebral artery (Fig. 1). There was extensive peri-rolandic damage, with lesion of the posterior aspect of the middle and inferior frontal gyri. Large sections of the left temporal lobe were lesioned, and damage extended to the left amygdala, corona radiata, lenticular nucleus and operculum. Within the left parietal lobe, damage extended to the anterior portion of the superior and inferior parietal lobules,