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Complex motion of shuttle buses in a transportation system reducing energy consumption

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ABSTRACT

We study the dynamical behavior and transitions of shuttle buses in a transportation system reducing energy consumption. We present the nonlinear-map model for the dynamics of M buses. The motion of shuttle buses depends on the loading parameter and the number. The dependence of the fixed points on the loading parameter is derived. The dynamical transitions occur at $2(M-1)$ stages with increasing the value of loading parameter. At the dynamical transition point, the motion of buses changes from a stable (an unstable) state to an unstable (a stable) state. The shuttle buses display periodic motions with various periods in the unstable state. In the unstable state, the number of riding passengers fluctuates complexly with varying trips.

1. Introduction

Recently, traffic congestion has attracted much attention in the fields of physics [1–5]. The jams are a typical signature of the complex behavior of traffic flow. Interesting dynamical phase transitions have been found in transportation systems. The jamming transitions have been studied for the traffic flow, pedestrian flow, and bus-route problems [6–28]. Also, the energy consumption in vehicular traffic has been investigated [29–32].

Until now, some models of a bus route system have been studied. In the bus route model with many buses, it has been found that the bunching transition between a heterogeneously jammed phase and a homogeneous phase occurs with increasing density [17–22]. In the cyclic bus system not passing each other, it has been shown that buses exhibit such complex behaviors as periodic and chaotic motions [27]. Also, it has been found that the distinct chaotic motion is induced by passing each other freely in the system including a few shuttle buses [28].

The shuttle bus system exhibits severe congestion problems in peak traffic. The maximum rate of serving passengers increases with the number of buses. In managing the shuttle bus operation, the usual criterion for deciding the number of buses is that one should be able to transport everyone from the starting point to his destination within some period of time for rush hour trips [26–28]. Another criterion used in shuttle bus operation is that a passenger's waiting time should not exceed some specified value. Furthermore, in a recent transport system of shuttle buses, it is necessary and important that the operator reduces energy consumption.

In real bus traffic, the inflow rate of passengers into buses varies highly with time. It is necessary to increase the number of buses with increasing the inflow rate. However, many buses are not necessary when the inflow rate decreases. In particular, in order to reduce energy consumption, it is important that the number of running buses varies with the inflow rate of passengers. However, there are few dynamical models to estimate the dynamic behavior of shuttle buses for the energy-saving transport system. It is important to estimate the tour time of buses.

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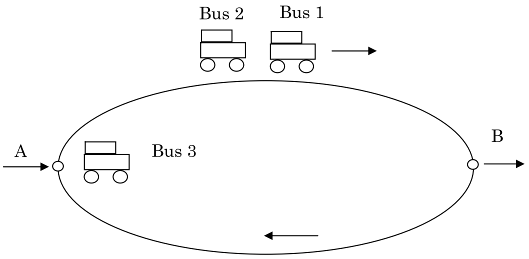

Fig. 1. Schematic illustration of shuttle buses. Point *A* is the starting point (origin) and point *B* is the destination. Passengers arrive continuously at the origin with a rate.

In this paper, we investigate the transportation of shuttle buses in an energy-saving system for reducing energy consumption. We present the nonlinear-map model of shuttle buses for the energy-saving system. We derive the fixed points of the nonlinear map. We show that the dynamical transitions to multi-periodic motion occur in bus traffic. We derive the dependence of the dynamical motion on the loading parameter and bus number M.

2. Nonlinear-map model

We consider the system of M buses shuttling between the starting point (origin) and the destination. The starting point is the only position to take the buses. The passengers board the buses at the origin, then the buses start at the origin, move toward the destination, all currently riding passengers leave the bus when the buses arrive at the destination, and the buses return to the origin. Fig. 1 shows the schematic illustration of shuttle buses. Point A is the starting point (origin) and point B is the destination. Passengers arrive continuously at the origin with a rate.

The shuttle bus system proposed here mimics the service of buses shuttling repeatedly between the airport and the railway station where the buses carry the passengers getting off the airplane to the railway station. This shuttle bus model has the realistic background.

First, we describe the dynamical model for the normal driving system in terms of the nonlinear map. In the normal driving system, M buses move together. We assume that passengers are distributed uniformly for M buses at the origin. Define the number of passengers boarding a bus at trip m by $B(m)/M$. The parameter γ is the time it takes one passenger to board the bus, so $\gamma B(m)/M$ is the amount of time needed to board all the passengers at the origin. The moving time of a bus is $2L/V$, where L is the distance between two terminals (the origin and the destination) and V is the mean speed of buses. The stopping time at the destination to leave the passengers is $\beta B(m)/M$ where parameter β is the time it takes one passenger to leave the bus. The tour time equals the sum of these periods. Then, the arrival time $t(m+1)$ of buses at the origin on trip $m+1$ is given by

$$t(m+1) = t(m) + \gamma B(m)/M + \frac{2L}{V} + \beta B(m)/M. \tag{1}$$

If the capacity of a bus is sufficiently large, all the passengers waiting at the origin can board the buses. New passengers arrive at the origin with rate μ. So $\mu(t(m) - t(m-1))$ is the number of passengers that have arrived since the previous buses left the origin. The number of passengers boarding the buses is expressed by

$$B(m) = \mu\left(t(m) - t(m-1)\right). \tag{2}$$

Then, we obtain the nonlinear map for the tour time

$$\Delta t(m+1) = \frac{(\gamma+\beta)\mu}{M}\Delta t(m) + \frac{2L}{V}, \tag{3}$$

where tour time $\Delta t(m)$ is defined as $\Delta t(m) = t(m) - t(m-1)$.

In the normal driving system, the buses can carry more passengers by increasing the number of buses. However, more energy is necessary to drive more buses even if the number of passengers decreases. So it is necessary and important to synchronize the number of buses with the number of passengers. In the energy-saving system, the number of buses reduces when the passengers decrease. Thus, one can reduce the energy consumption when the number of passengers varies from time to time. The upper limit of passengers is proportional to the number of buses. We propose the nonlinear-map model for the energy-saving traffic system. The number of buses increases one by one when the tour time is superior to the prescribed values $t_{i-1,i} (i = 1, 2, \ldots, M)$, while the number of buses decreases one by one if the tour time is less than the prescribed values. Fig. 1 shows the schematic illustration of the energy-saving system of three buses. Buses 1 and 2 move together and bus 3 stops at the origin. The operator controls the number of buses at the origin. If the inflow rate is higher, bus 3 runs. When the inflow rate is less, bus 2 stops at the origin. The tour time for the energy-saving system is given by

$$\Delta t(m+1) = (\gamma+\beta)\mu\Delta t(m) + \frac{2L}{V} \quad \text{for } \Delta t(m) < t_{1,2},$$