Information Sciences 222 (2013) 384–404

Contents lists available at SciVerse ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

How to handle uncertainties in AHP: The Cloud Delphi hierarchical analysis

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ARTICLE INFO

Article history:
Received 11 January 2011
Received in revised form 12 August 2012
Accepted 14 August 2012
Available online 29 August 2012

Keywords:
Multi-criterion group decision making
Decision making under uncertainty
Cloud Delphi hierarchical analysis
Analytic hierarchy process
Cloud model

ABSTRACT

In practice, many practical problems occur in uncertain environments, especially in situations that involve human subjective evaluation such as that in the analytic hierarchy process (AHP). This paper presents a practical multi-criteria group decision-making method for decision making under uncertainty. To handle the randomness and fuzziness of individual judgments, the normal Cloud model, group decision-making technique, and the Delphi feedback method are adopted. In the proposed Cloud Delphi hierarchical analysis (CDHA), experts are asked to express their judgments using interval numbers. Individual fuzziness and randomness are then mined from the interval-value comparison matrices. Subsequently, the interval-value pairwise comparison matrices are converted into the corresponding Cloud matrices, and the one-iteration Delphi process is executed to diminish individual judgment mistakes. The individual Cloud weight vectors are calculated using the geometric mean technique and are finally weighted to form the group Cloud weight vector. A simple case study that involved reproducing the relative area sizes of six provinces in China shows that the CDHA method can effectively reduce mistakes and improve decision makers' judgments in situations that require subjective expertise and judgmental inputs. In addition, a practical decision-making problem in which houses are ranked by home buyers shows that the proposed method is effective when applied to complex, large, multidisciplinary problems with considerable uncertainties.

1. Introduction

1.1. AHP and fuzzy AHP

Analytic hierarchy process (AHP) [28–30] is one of the most commonly used utility-based methods for multi-criteria (-attribute) decision making [41]. The AHP uses objective mathematics to process the subjective and personal preferences of an individual or a group in decision making. In Saaty's hierarchical analysis, a person (expert, judge, etc.) is asked to provide his/her ratios a_{ij} for each pairwise comparison between issues (alternatives, candidates, etc.) $A_l, A_2, \ldots, A_n$ for each criterion (objective) in a hierarchy and also between the criteria. To make comparisons, a scale of numbers should be used to indicate how many times more important or dominant one element is over another element with respect to the criterion or property being compared. In Saaty's fundamental nine-scale measurement used in making a comparison, the numbers for the ratios are usually taken from the set $\{1, 2, \ldots, 9\}$. It consists of verbal judgments that range from equal to extreme (equal, moderately more, strongly more, very strongly more, extremely more). The numerical judgments (1,3,5,7,9) correspond to

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http://dx.doi.org/10.1016/j.ins.2012.08.019

the verbal judgments and compromises between these values. For example, if a person considers A_1 to be moderately more important than A_2, then a_{l2} is equal to 3/l. The ratio a_{ij} indicates the strength with which A_i dominates A_j. The nine-scale measurement is widely applied in AHP. However, the language description disagrees with the numerical values of the scale division in various aspects [47].

The first disagreement is that the numerical values are not exactly equivalent to the verbal judgments. If the sizes (usually importances) of a_l and a_2 are compared, $a_{l2} = 3$ represents that a_l is three times as large (important) as a_2. However, the corresponding verbal judgment states that a_l is moderately larger (moderately more important) than a_2. Usually, in our opinion, "three times as large" does not merely indicate moderately more importance. Furthermore, if a_l is moderately more important than a_2, and a_2 is moderately more important than a_3, a_l might be considered more or strongly more important than a_3. However, in Saaty's nine-scale measurement, $a_{l3} = a_l/a_3 = (a_l/a_2) \times (a_2/a_3) = 3 \times 3 = 9$, which means that a_l is extremely more important than a_3. This finding reflects an inconsistency between the qualitative descriptions and their corresponding quantitative numbers. In addition, the intrinsically nonlinear subjective perceptions of human beings usually lead to conflicts and inconsistencies in AHP.

The second disagreement is that the same qualitative verbal judgment has different meanings for different persons (experts, judges, etc.). One person may say that "moderately more" represents 1.2 or so, but to another person, it represents 2 or so. Sometimes, even the same person (expert, judge, etc.) may assign different meanings to the same qualitative verbal judgment in different situations. For example, in the three circles in Fig. 1a, one may say that A is moderately larger than B, but if presented with Fig. 1b, the same person may think A is strongly larger than B. However, the two As and the two Bs in Fig. 1a and b are the same. One can obtain different qualitative verbal judgment results even with the same A and B in different situations.

Ensuring consistency/unison in AHP and in group decision is difficult because of the drawbacks of qualitative linguistic judgments, particularly the disagreements between the language description and the numeric relation of the nine-scale division measurement. In practice, establishing uniform linguistic term sets for different people and problems is almost impossible. Thus, a unitive scale measurement like crisp numbers should be used to conduct comparisons. However, assigning pairwise comparisons usually involves uncertainties because of the inherent subjective nature of human judgments.

The complexity and uncertainty involved in real-world decision-making problems and the inherent subjective nature of human judgments pose challenges for experts in developing crisp decision-making methodologies with precise numerical values. Decision-making processes also become difficult to implement. Providing fuzzy or interval-value opinions for the judgments in a pairwise comparison matrix is easier, and a number of techniques that use a fuzzy or interval comparison matrix to generate weights have been developed.

The earliest work in fuzzy AHP was conducted by Van Laarhoven and Pedrycz [37]. In their work, the triangular fuzzy number of the fuzzy set theory was brought directly into the pairwise comparison matrix of the AHP. Buckley [4] used fuzzy numbers instead of exact (crisp) numbers and utilized the geometric mean method to calculate fuzzy weights. In the proposed fuzzy hierarchical analysis (FHA), ratio a_{ij} is expressed as approximately 5/1 instead of exactly 5/l, or that a ratio is between 4/1 and 6/1 instead of exactly 5/l. The membership functions for the final fuzzy weights of FHA can be shown graphically, allowing an intuitive ranking of the alternatives. However, the membership functions are nonlinear, and clear parameters that have certain physical meanings to denote the fuzziness and uncertainty of the final fuzzy weights do not exist. Chang [6] introduced a new approach for handling fuzzy AHP using triangular fuzzy numbers for pairwise comparisons and the extent analysis method for the synthetic extent values of the pairwise comparisons. Chang's fuzzy AHP derives crisp weights for fuzzy comparison matrices. The computational simplicity of this approach has attracted a number of applications. However, Wang et al. [39] recently proved that such a method was found to be unable to derive the true weights from a fuzzy or crisp comparison matrix. Moreover, the weights determined using the extent analysis method did not represent the relative importance of decision criteria or alternatives at all. Buckley et al. [5] directly fuzzified Saaty's original procedure of computing weights in hierarchical analysis to obtain fuzzy weights in the FHA. However, solving a series of (they used α -cuts) nonlinear optimization fuzzy models is computationally complicated because these models usually require intelligent algorithms such as evolutionary algorithm [5] and integrate simulated annealing algorithm, neural network, and fuzzy simulation techniques [22]. Csutora and Buckley [11] developed the lambda-max method, which is the direct fuzzification of the well-known λ_{max} method. This method also entails transforming a fuzzy comparison matrix into a series of interval comparison matrices using α -level sets and the extension principle. Therefore, the method requires the solution of a series of eigenvalue problems. Wang et al. [40] proposed an eigenvector method (EM) to derive a normalized interval or fuzzy

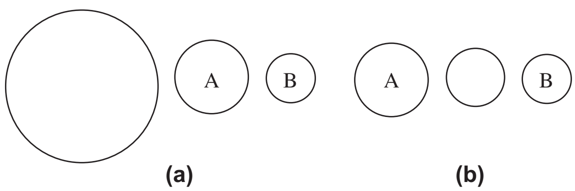

Fig. 1. Comparisons of A and B in different situations.