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Synthesis of individual best local priority vectors in AHP-group decision making

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ABSTRACT

An assessment of the individual judgments and AHP-produced priority vectors for involved decision-makers indicates that the individual consistencies of decision makers may vary significantly, thus making the final group decision less reliable. In this paper, an approach is proposed as to how to combine decision makers' local priority vectors in AHP synthesis and reduce so-called group inconsistency. Instead of aggregating individual judgments (AIJ), or aggregating individually derived final priorities (AIP), we propose to perform an AHP synthesis of the best local priority vectors taken from the most consistent decision makers. The approach and related algorithm we label as MGPS after the key terms 'multicriteria group prioritization synthesis.' The concept is analogous to the one proposed by Srdjevic [1] for individual AHP applications where the best local priority vectors are selected based on the consistency performance of several of the most popular prioritization methods. Here, decision makers are combined instead of prioritization methods, and group context is fully implemented. After completing an evaluation of the decision makers inconsistencies in each node of the hierarchy, the selected best local priority vectors are synthesized in a standard manner, and the final solution is declared to be an AHP-group decision. Two numerical examples indicate that the developed approach and algorithm generate the final priorities of alternatives with the lowest overall inconsistency (in the multicriteria sense).

1. Introduction

Group decision making problems arise from many real-world situations in many fields such as human spaceflight mission planning, water management, and selection of advanced technology (Choudhury et al. [2], Srdjevic [3], Tavana and Hatami-Marbini [4]). In recent times, most of these problems are attacked and successfully solved with the application of the analytic hierarchy process (AHP), developed by Saaty [5]. In fact, the so-called AHP-group application means that the standard AHP-individual application is extended to provide for certain types of aggregations, consensus procedures, etc. Rich, pertinent literature in this regard is referenced appropriately in the remaining part of this text.

Three mainstream theories and applications comprise AHP. The first theory applies to the preference relations - linguistic, numerical, and fuzzy (e.g. Saaty [5], Chiclana et al. [6], Fu and Yang [7], Herrera and Martínez [8], Herrera et al. [9]) - that are widely used in individual and group decision making. The second one applies to the prioritization methods for extracting cardinal information (weights of the decision elements, usually the criteria and alternatives) from so-called judgment matrices (i.e. multiplicative preference relations) at each node of a hierarchy. The third mainstream theory applies to consensus building, aggregation methods, and measuring (in)consistency in group decision making. This paper falls within this third theory and focuses on the AHP-group synthesis of the best local priority vectors identified by multicriteria evaluation of inconsistency contained in individual vectors obtained for participating members of a group. To our best knowledge, the approach we propose is novel and the related computational procedure is labeled as MGPS algorithm, after the key terms 'multicriteria group prioritization synthesis.'

The AHP offers a variety of options in supporting the decision making processes, including group contexts. Being a soft computing technique as well as a multicriteria analysis method, the proposed approach is aimed to contribute to the soft computing community by introducing an objective method of synthesizing locally computed priority vectors for all involved individuals in a group. In general, computations within AHP are inherently simple. However, an issue of (in)consistency in decision makers in modern times presents the challenge of employing evolution strategies, genetic algorithms, particle swarm optimization and other soft computing techniques and heuristics in deriving priorities from inconsistent (or low consistency) matrices. In a way, a new paradigm aims to provide more efficient solving of the spectrum of decision making problems where individual judgments of decision makers play a leading role. The MGPS algorithm we propose is inspired by an idea for making objective the process of prioritizing alternatives vs. a global goal. We developed it to be straightforward and easy

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to program and implement in real life applications. Being fully programmed in FORTRAN, on a proof-of-concept level it was successfully used in many test applications as well as in two described examples presented in the second part of this paper.

Most of the research papers dominantly report theoretical works and application results concentrating on just one ratio-scale matrix at a time. Much fewer research reports relate to the whole hierarchy where the original AHP philosophy belongs to and the concept of AHP synthesis is fully implemented. This paper relates to the latter case only; that is, we consider a complete AHP hierarchy, not just one local matrix. In addition, the presented work is aimed to cover several topics related to AHP application in group contexts where decision makers demonstrate different consistencies.

The core of our approach is to perform AHP synthesis of the best local priority vectors taken from the most consistent decision makers. Without losing generality, if we consider a three level hierarchy (goal-criteria-alternatives), the number of matrices is $1 + na$, where na is the number of alternatives. If there are K decision makers then there are $K(1 + na)$ matrices, and in every node of a hierarchy there is a set of K matrices with their priority vectors. Some vectors are more consistent than others, and it is possible to identify the best one if certain criteria are adopted for assessing their quality regarding consistency. On the other hand, consistency measures applicable to AHP can be divided into two groups: (a) general (e.g. Euclidean Distance or Minimum Violation), and (b) specific (e.g. prioritization method related, such as *CR* for eigenvector method or *GCI* for logarithmic least square method). If consistency measures are used as criteria for assessing the quality of priority vectors derived from decision makers at a given node of hierarchy, then the best (optimal in multicriteria sense) priority vector can be identified and propagated to the final AHP synthesis. Obviously, the final synthesis is performed with the best vectors and therefore the final (group) priorities of alternatives are objectively the best possible.

The paper is organized as follows: in Section 2, we briefly present reviewed related research, basic preliminary knowledge of AHP, including its most commonly used prioritization method known as the eigenvector (EV) method. In Section 3, we develop an approach to AHP-group synthesis following an idea presented in Srdjevic [1], and instead of AIJ or AIP aggregations, we propose to combine the local priorities derived from group members based on a multicriteria evaluation of their demonstrated consistencies. In Sections 4 and 5 two illustrative examples from real-life AHP-group applications are provided, and the results of MGPS application are discussed. Section 6 presents concluding remarks and an agenda for future research in the subject area.

2. Related research

2.1. AHP

AHP (Saaty [5]) is one of the most popular decision support tools because of its powerfulness, simplicity, and potential of being utilized for the group decision-making process that involves multiple actors, scenarios, and decision elements (criteria, sub criteria, and alternatives). As correctly stated by Altuzarra et al. [16], AHP is 'one of the methods that best captures changes in philosophy (from mechanistic reductionism to evolutionist holism), methodology (from the search for truth to the search for knowledge), and technology (communication networks) that took place in the latter years of the 20th century.'

The AHP requires a well-structured problem represented as a hierarchy with the goal at the top. The subsequent levels contain the criteria and sub-criteria, while alternatives lie at the bottom of the hierarchy. The AHP determines the preferences among the set of alternatives by employing pairwise comparisons of the hierarchy elements at all levels following the rule that at a given hierarchy level, elements are compared with respect to the elements in the higher level by using the fundamental importance scale (Saaty [5]).

The synthesis is performed by multiplying the criteria-specific priority vector of the alternatives with the corresponding criterion weight and summing up the results to obtain the final composite of the alternatives' priorities with respect to the goal. The highest value of the priority vector indicates the best-ranked alternative.

2.2. Prioritization in AHP

Over the years, several methods have been proposed for estimating the weights from a matrix of pairwise comparisons, including additive normalization (AN), eigenvector (EV), logarithmic least squares (LLS), weighted logarithmic least square (WLS), logarithmic goal programming (LGP), fuzzy preference programming (FPP), and others. A brief description of these competing methods is provided by Harker and Vargas [10], and Srdjevic [1]. Herein, we present the main features of the EV method because we consider it to be competitive with the other methods and it is commonly used in practice as well as in our research.

The EV method generates a priority vector for the given pairwise comparison matrix obtained from the decision maker. The method, originally proposed by Saaty [5], solves an eigenvalue problem associated with a matrix of size n and is nicely described in Chandran et al. [11]. The mathematical notation that follows is introduction to the next section where the MGPS algorithm will be described in detail.

Let $A = (a_{ij})$, for $i, j = 1, 2, \ldots, n$, denote a square pairwise comparison matrix, where entry a_{ij} gives the importance of element i relative to element j. Each entry is a positive value ($a_{ij} > 0$) with a reciprocal $a_{ji} = 1/a_{ij}$ for all $i, j = 1, 2, \ldots, n$). The decision maker wants to compute a vector of weights $(w_1, w_2, \ldots, w_n)$ associated with A.

If the matrix $\boldsymbol{A}$ is consistent (that is, $a_{ij} = a_{ik}a_{kj}$ for all $i, j, k = 1, 2, \ldots, n$), then $\boldsymbol{A}$ contains no errors. Therefore, the weights are already known, and we have

$$a_{ij} = \frac{w_i}{w_j}, \quad i, j = 1, 2, \ldots, n. \tag{1}$$

Summing over all j, we obtain

$$\sum_{j=1}^{n} a_{ij} w_j = n w_i, \quad i = 1, 2, \ldots, n. \tag{2}$$

In matrix notation, the result is equivalent to

$$\boldsymbol{A}\boldsymbol{w} = n\boldsymbol{w}, \quad \boldsymbol{e}^T\boldsymbol{w} = 1 \tag{3}$$

The vector $\boldsymbol{w}$ is the principal right eigenvector of matrix $\boldsymbol{A}$ corresponding to the eigenvalue n. If the vector of weights is not known, then it can be estimated from the pairwise comparison of matrix $\boldsymbol{A}'$ generated by the decision maker and solving

$$\boldsymbol{A}'\boldsymbol{w}' = \lambda'\boldsymbol{w}', \quad \boldsymbol{e}^T\boldsymbol{w}' = 1 \tag{4}$$

for $\boldsymbol{w}'$. The matrix $\boldsymbol{A}'$ contains the pairwise judgments of the decision maker and approximates the matrix $\boldsymbol{A}$ whose entries are unknown. In Eq. (4), λ' is an eigenvalue of $\boldsymbol{A}'$, and $\boldsymbol{w}'$ is the estimated vector of weights. Saaty [5] uses the largest eigenvalue λ_{max} of $\boldsymbol{A}'$ when solving for $\boldsymbol{w}'$ in

$$\boldsymbol{A}'\boldsymbol{w}' = \lambda_{\max}\boldsymbol{w}', \quad \boldsymbol{e}^T\boldsymbol{w}' = 1 \tag{5}$$

Saaty has shown that λ_{max} is always greater than or equal to n, and if its value is close to n then the estimated vector of weights $\boldsymbol{w}'$ solves Eq. (3) approximately.

A good estimate of the principal eigenvector for an inconsistent matrix is obtainable by consecutively squaring the matrix, normalizing the row sums each time, and stopping the procedure when