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A two-phase algorithm for consensus building in AHP-group decision making

Bojan Srdjevic *, Zorica Srdjevic, Bosko Blagojevic, Kosana Suvocarev

Group for Systems Analysis and Decision Making, Department of Water Management, Faculty of Agriculture, University of Novi Sad, Trg D. Obradovica 8, 21000 Novi Sad, Republic of Serbia

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ABSTRACT

Group decision making through the AHP has received significant attention in contemporary research, the primary focus of which has been on the issues of consistency and consensus building. In this paper, we concentrate on the latter and present a two-phase algorithm based on the optimal clustering of decision makers (members of a group) into sub groups followed by consensus building both within sub groups and between sub groups. Two-dimensional Sammon's mapping is proposed as a tool for generating an approximate visualization of sub groups identified in multidimensional vector space, while the consensus convergence model is suggested for reaching agreement amongst individuals in and between sub groups. As a given, all decision makers evaluate the same decision elements within the AHP framework and produce individual scores of these decision elements. The consensual scores are obtained through the iterative procedure and the final scores are declared as the group decision. The results of two selected numerical examples are compared with two sets of results: the results obtained by the commonly used geometric mean aggregation method and also the results obtained if the consensus convergence model is applied directly without the prior clustering of the decision makers. The comparisons indicated the expected differences among the aggregation schemes and the final group scores. The matrices of respect values in the consensus convergence model, obtained for cases when the decision makers are optimally clustered and when they are not, show that in the latter case the decision makers receive lower weights of respect from other members in the group. Various tests showed that our approach is efficient in cases when no clusters can be visually and undoubtedly identified, especially if the number of group members is high.

1. Introduction

The Analytic Hierarchy Process (AHP) [1] is a widely used multi-criteria tool in group decision making. It involves the structuring of a decision problem as a hierarchy and pair-wise comparisons between decision elements to obtain their weights of importance. The final result is a hierarchy with the vector of alternatives' weights at the bottom level and a goal at the top hierarchy level.

The philosophy of AHP is to synthesize in a downward direction the weights computed from local pair-wise comparison matrices created by the decision maker (DM) at all levels of a hierarchy. The DM can be a personalized individual or a virtual individual representing a group of personalized individuals. In either case, AHP assumes dealing with a hierarchy of at least

* Corresponding author. Tel.: +381 21 4853 337.
E-mail addresses: srdjevicz@polj.uns.ac.rs, bojans@polj.uns.ac.rs (B. Srdjevic).

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three levels (goal-criteria-alternatives) and a weighted synthesis of local vectors until the bottom level containing the alternatives is reached. The final vector has entries being global weights of alternatives with respect to a goal.

At the local level of a hierarchy, one comparison matrix is considered at a time. By using a certain ratio scale, i.e. Saaty's 9-point scale, the DM mutually compares decision elements. Any of the prioritization methods described in Ref. [2] can be employed to derive the weights of compared elements and consider these weights as entries of the so-called local priority vector. Unless a complete hierarchy is considered, the described process can be labeled as 'deriving priorities of decision elements in the manner of AHP'. Only the synthesis of more local priority vectors, which actually occurs if there are at least three levels in a hierarchy, deserves to be declared as 'deriving priorities of alternatives by the AHP'. In both cases it is common that at a given stage there will be a computed priority vector $\boldsymbol{w} = (w_1, w_2, \ldots, w_n)$ containing the weights of the compared n decision elements, and this is a starting point for the proposed two-stage algorithm for building consensus in AHP-group decision making.

The group decision making context implies that any group or sub group of DMs can set the weights of elements in a hierarchy through compromise, consensus, voting, direct weighted arithmetic or geometric mean aggregation, individual judgments or priorities, or by other such means. Any of these methods can be time consuming and can produce different final results, or, in some cases, may end-up without a final decision. On the other hand, well-known weighted arithmetic and geometric aggregation schemes require prior definition of the weights of importance of the members in a group and may lead to a final result which the participants do not feel to be their own. Assigning weights to the participants is even more problematic if there are many of them.

An option in group decision making is to derive the final group decision by building a consensus, classically defined as the full and unanimous agreement of all the DMs regarding all the possible alternatives [3]. There are formal mathematical methods for reaching the consensus. Inspired by Chiclana et al.'s [4] framework for integrating individual consistency into consensus models, Dong et al. [5] developed two AHP consensus models under the row geometric mean method [6]. The consensus indices are defined to measure the degree of consensus among judgment matrices (or DMs) and simulation experiments are presented which show that the proposed models can improve the consensus indices of judgment matrices and help AHP decision makers reach a consensus. It is argued that their proposal has two desired features: firstly, that in reaching a consensus, the adjusted judgment matrix has a better individual consistency index (i.e. geometric consistency index) than the corresponding original judgment matrix; and secondly, that their proposal satisfies the Pareto principle of social choice theory. Both arguments, in our opinion, deserve continued discussion. Namely, our experiments with Dong et al.'s [5] consensus model showed that adjustments of judgment matrices for larger groups of decision makers can be very time consuming. That is, the convergence process is very slow because of too many iterations. On the other hand, satisfaction of the Pareto principle in an AHP-group context is an issue that is weakly connected with the social choice theory, which also deserves attention in future research.

There are other mathematical models aimed to build consensus of individuals participating in the decision making group, such as consensus convergence modeling or central tendency methods (middle 'value' is measured using the mean, median or mode). One of the best known formal models is the consensus convergence model [7], where, by repeatable mathematical procedure and through a mutual respect based on local or final priority vectors of decision elements, the decision makers do not only achieve consensus on the issue under consideration but also agree on the overall relative weight of each group member. This model is considered as a suitable conflict resolution method in management problems because its mathematical structure captures a typical situation of disagreement, and refusing to change one's opinion would, in mathematical terms, be equivalent to assigning a null weight to all other members and full weight to one's self [8]. However, this situation is one of pure dogmatism that is unacceptable in modern decision making.

In reality, a group is usually heterogeneous, i.e. composed of DMs different in age, gender, attitudes, interests and/or knowledge regarding the problem at hand. Although diversity stimulates creativity and leads in turn to improved decision making, making a true decision becomes a difficult task and is sometimes confusing due to that same diversity. If DMs provide more or less homogeneous responses, decision making becomes more realistic and consistent [9]. There are many different approaches employed in order to cluster DMs, recently based on data envelopment analysis [10], genetic algorithms [11], variable precision rough set model [12], and fuzzy sets [9,13].

Although it would aid in identifying optimal decision groups to visualize clusters of DMs identified within the decision group, only a few papers concentrate on this issue [3,14–16]. For example, in Condon et al. [3] Sammon's mapping is used to cluster eight DMs from the government agency in order to recognize outliers and to encourage DMs to be forthcoming and objective.

The major objective of this paper is to point out how information obtained by the decision makers, namely their local or final priority vectors, can be aggregated, i.e. used to reach the consensus based group decision. We propose to gather DMs into an optimal number of clusters (sub groups) by using their individually computed priority vectors at a given level of the AHP hierarchy. If the number of entries in a vector is n, an optimal clustering should be performed in original n-dimensional (nD) vector space rather than in 2D space, which is possible, e.g., if Sammon's mapping is employed. Given optimally created sub groups of DMs, the final group decision (the group priority vector) is derived by the consensus convergence model in two phases: firstly, by reaching consensus within each sub group, and secondly, by reaching consensus between sub groups.

The paper is organized as follows. In Section 2, preliminary knowledge on AHP is briefly presented, followed by a short description of the main features of the multidimensional optimal clustering we used in our approach and two dimensional (Sammon's) mapping, which is convenient and especially useful for visualizing positions of DMs and their clusters. The