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Discrete Optimization

Network cost minimization using threshold-based discounting

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Abstract

A network design problem in which every pair of nodes can communicate directly is discussed. However, there is an incentive to combine flow from different sources, namely, if the total flow through a link exceeds the prescribed threshold, then the cost of this flow is discounted by a factor α. Alternative mixed integer linear formulations for this problem are presented. Computational results comparing the models on a set of benchmark problems are also presented. The results show the effectiveness of the formulations: for discounts of 5–10%, the gaps between linear and integer solutions are within few percent. Such a model offers economic incentives in building and utilizing communication networks.

Keywords: Mixed integer programming; Branch and bound; Hubs; Network design

1. Introduction

Special type of networks, so-called *hub-and-spoke* networks, have been used by the airline and telecommunication industry. A hub-and-spoke network supports traffic flows between multiple origins and destinations (*many to many* network). In order to exploit economies of scale, smaller number of links are selected to serve a large number of origin–destination pairs.

Nodes where amalgamation is taking place (*hubs*) play a role of central transshipment facilities. The subnetwork consisting only of hub nodes is completely interconnected. Utilizing hubs as transshipment facilities implies a reduction in construction costs, centralized handling and sorting, and allows carriers to take advantage of economies of scale. There are many different instances of the hub location problem (HLP), and a common thread is to decide on which locations the investor must establish hubs in order to minimize the cost of the total traffic flow. The incentive for amalgamation of flow is offered through

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discounted costs for interhub traffic. This problem has been addressed in many papers.

A detailed overview of existing formulations for p-HLPs can be found in Campbell [2]. Integer programming formulations for p-hub median problem, uncapacitated HLP, p-hub center problem and hub covering problems are presented. Several variations of these problems include the concept of flow thresholds on hub-spoke links. In this case, a link can be established only when the flow exceeds the threshold value.

Skorin-Kapov et al. [16] have developed tight LP relaxations for hub location and some related uncapacitated p-hub median problems. Their approach led to exact solvability for the Civil Aeronautical Board (CAB) benchmark data set (air-traffic between 25 major US cities).

Ernst and Krishnamoorthy [5] proposed a redesign of a postal delivery network in order to minimize overall costs. Services in their network can be sorted into three types: collection, transfer and distribution. Unit costs associated with these activities are proportional to distances. The cost factor for transfer (α) is a discount factor offered for inter-hub traffic, i.e., $\alpha < 1$. Four and three dimensional formulations were introduced and computationally compared (size vs. tightness). A procedure of adding cuts to tighten the formulations was developed. They used a data set generated by the Australia Post (AP) that contains 200 nodes representing postal districts. Smaller subproblems were used for the analysis of the behavior of shortest path-based heuristic, enumeration, and branch and bound algorithms.

In [7] a network design problem was presented based on three service policies (one-stop, two-stop, all-stop). No hub-and-spoke structure was assumed. The objective was to minimize costs of running a single airline with several types of aircraft in its fleet. For all the policies, a heuristic solution procedure based on local improvements was developed and tested. From the obtained solutions they concluded that several nodes might be strong candidates for hub locations regardless of the policy used. The location of the candidates depended more on their geographical position than on their demand level. The tests were performed on the CAB data set and on a 39-node data set generated by a simple gravity model.

An analysis of hub-to-hub link utilization might reveal that some links are indeed very busy. However, it can happen that the rest of the hub-to-hub links carry only a small amount of flow. Nevertheless, the cost of this small traffic is discounted because it is going through the hub-to-hub link. The amounts of traffic on the discounted links can be disproportional, which brings the idea of discounting only heavily used links. In [12] the HLP was modified to include possibilities of differential discounts on interhub links, depending on total traffic amounts, namely, an interhub link with larger flow will allow a larger discount than some other, less heavily used interhub link. The inherently nonlinear cost function is approximated by a piecewise continuous linear function, hence allowing linear programming solution methods. The cost function is increasing at decreasing rate as flows increase. Such an approach offers a more realistic treatment of discounts available to network users.

Even in the case where the cost function is piecewise linear, the search for the optimal solution might be very difficult. One such class of problems, where the cost of a link is a concave nondecreasing piecewise linear function, is known as the *Minimum Concave Cost Network Flow* problem (MCCNF). The MCCNF problem is known to be NP-hard, and several solution techniques for the problem have been developed over the last two decades. One of the specialized MCCNF solution techniques is presented in [10]. One can also refer to [6].

Bryan [1] provides some additional extensions to hub location problems designed to increase efficiency of interhub links. In one of her formulations interhub links with flow below a minimum threshold are not open. In yet another formulation, the hubs should be completely interconnected. However, it is required that interhub links achieve a certain level of flow. This requirement determines the number of hubs to be open. The location of hubs and allocation of non-hub nodes is simultaneously provided.

In this paper, we develop a model outside the traditional hub approach by emphasizing the