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An exact algorithm for cost minimization in series reliability systems with multiple component choices ☆

Ning Ruan, Xiaoling Sun *

Department of Mathematics, Shanghai University, Baoshan 200444, Shanghai, PR China

Abstract

In this paper, we present an exact method for cost minimization problems in series reliability systems with multiple component choices. The problem can be modelled as a nonlinear integer programming problem with a nonseparable constraint function. The method is of a combined Lagrangian relaxation and linearization method. A Lagrangian bound is obtained by solving the dual of a separable subproblem. An alternative lower bound is derived by 0–1 linearization method. A special cut-and-partition scheme is proposed to reduce the duality gap, thus ensuring the convergence of the method. Computational results are reported to show the efficiency of the method.

Keywords: Reliability system; Cost minimization; Multiple component choice; Lagrangian bound; Branch-and-bound method

1. Introduction

Reliability optimization problems are often encountered in many industrial and engineering applications. One of the popular techniques in improving the reliability of a series system is to use parallel redundancy. Fig. 1 illustrates the structure of the series–parallel network.

The components in Fig. 1 may represent electronic parts in a section of circuits, coolers and filters in a lubrication system, valves in a pipeline (see, e.g., [1,21,24]) or subsystems of a complicated communication networks.

In this paper, we consider the cost minimization problem in series system with multiple component choices. The problem is to minimize the cost of a series system under a minimum overall reliability requirement. The problem can be modelled as the following nonlinear integer programming:

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* Corresponding author.
E-mail address: xlsun@staff.shu.edu.cn (X. Sun).

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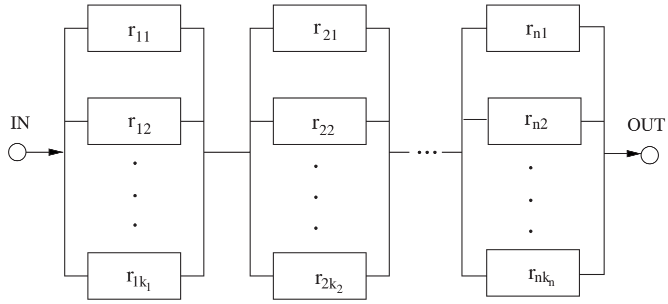

Fig. 1. Diagram of a series system with multiple component choices.

$$
\begin{aligned}
(\mathrm{P}) \quad \min \quad & c(x)=\sum_{i=1}^{n}\sum_{j=1}^{k_i} c_{ij}(x_{ij}) \\
\text{s.t.} \quad & R(x)=\prod_{i=1}^{n}\left[1-\prod_{j=1}^{k_i}(1-r_{ij})^{x_{ij}}\right] \geqslant R_0, \\
& x \in X=\{x \mid 0 \leqslant x_{ij} \leqslant u_{ij}, x_{ij} \text{ integer}, \ i=1,\ldots,n, j=1,\ldots,k_i\},
\end{aligned}
$$

where

- $r_{ij} \in (0,1)$: the reliability of the jth component (choice) in the ith subsystem in series;
- x_{ij}: the number of identical redundancy of the jth components in the ith subsystem;
- u_{ij}: the upper bound of the identical redundancy of the jth components in the ith subsystem;
- $c_{ij}(x_{ij})$: a convex and increasing function of x_{ij} represents the cost of having x_{ij} identical jth component in the ith subsystem;
- $1-\prod_{j=1}^{k_i}(1-r_{ij})^{x_{ij}}$: the reliability of the ith subsystem;
- $R(x)$: the overall system reliability when adopting redundancy assignment

  $$x=(x_{11},\ldots,x_{1k_1},\ldots,x_{n1},\ldots,x_{nk_n})^{\mathrm{T}};$$
- $R_0 \in (0,1)$: a given minimum reliability level.

Since r_{ij}'s can be different in the ith subsystem, the reliability function $R(x)$ is in general a nonseparable function. This makes it a great challenge to design efficient solution methods for (P). It is noticed that the well studied *simple* series–parallel system is a special case of problem (P) when $r_{i1}=\cdots=r_{ik_i}$ for $i=1,\ldots,n$ (see [5,19,24,25]).

Exact solution methods in reliability optimization are mainly for simple series–parallel system. For example, branch-and-bound methods and its combinations with dynamic programming methods, various partial enumeration techniques (see [11,14,17]) and cutting plane method [12]. Few implementable methods have been developed in the literature for reliability optimization problem with a nonseparable reliability function. Misra and Sharma [13] proposed a search algorithm to scan the entire feasible region of the optimal redundancy problem (see also [16]). Ng and Sancho [14] proposed a dynamic programming method combined with depth-first search technique. Chern and Jan [3] presented a two-phase method for solving the reliability problem. Sung and Cho [22,23] transformed the reliability problem into an equivalent binary integer problem and proposed a solution space reduction procedure to improve the algorithm. There are also heuristic methods that search for a near-optimal solution of reliability optimization problems. For example, genetic algorithms [4,6] and greedy methods [9] .

In this paper, we propose a new exact method for solving problem (P). This method is based on the Lagrangian dual search and linearization method. To overcome the nonseparability of the constraint, we first approximate $R(x)$ by a linear function. Lagrangian bounds of the approximation problem can be obtained by