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The role of repair strategy in warranty cost minimization: An investigation via quasi-renewal processes

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ABSTRACT

Most companies seek efficient rectification strategies to keep their warranty related costs under control. This study develops and investigates different repair strategies for one- and two-dimensional warranties with the objective of minimizing manufacturer's expected warranty cost. Static, improved and dynamic repair strategies are proposed and analyzed under different warranty structures. Numerical experimentation with representative cost functions indicates that performance of the policies depend on various factors such as product reliability, structure of the cost function and type of the warranty contract.

1. Introduction

Extensive warranties are commonly offered by a wide range of manufacturers as a means of survival in increasingly fierce market conditions. Faced with the challenge of keeping the associated costs under control, most companies seek efficient rectification strategies. In this study, different repair strategies are developed and investigated under one- and two-dimensional warranties with the intent of minimizing the manufacturer's expected warranty cost. Quasi-renewal processes are used to model the product failures along with the associated repair actions. Based on quasi-renewal processes, three different repair policies – static, improved and dynamic – are proposed, and representative cost functions are developed to evaluate the effectiveness of these alternative policies.

In a one-dimensional warranty, the warrantor agrees to rectify or compensate the customer for the failed items within a certain time limit after time of sale. A two-dimensional warranty is a natural extension where the warranty period is characterized by a region defined simultaneously by time and usage. Examples of two-dimensional warranties are widely seen in the automotive industry where vehicles are covered under warranty until a certain age or mileage after the initial purchase.

Karim and Suzuki (2005) provide a recent survey of the literature on statistical models and methods for warranty analysis. They present a summary of important mathematical findings such as estimators of critical parameters used in the analysis of warranty claim data. Thomas and Rao (1999) and Murthy and Djamaludin (2002) are also important review papers on product warranty. Thomas and Rao (1999) adopt a management perspective and focus on the works that address quantification of warranty costs and determination of warranty policies. They also present some research directions. Murthy and Djamaludin (2002) follow a broader perspective. They build on Murthy and Blischke's (1992a,b) paper and cover the pertinent academic developments in the areas of cost analysis, engineering design, marketing, logistics and management systems. They also mention applications in some other related areas such as law, accounting, economics and sociology.

Of particular interest for the current study is the modeling of rectification actions in the warranty context. Majority of the literature on one- and two-dimensional warranties considers perfect and minimal repairs. Imperfect repair is widely modeled as a combination of perfect and minimal repair. Barlow and Hunter (1960) are the first to combine the perfect and minimal repair under one-dimensional warranties. The studies of Cleroux et al. (1979), Boland and Proschan (1982), Phelps (1983) and Nguyen and Murthy (1984) give some other examples of combination repair/replace models under one-dimensional warranty. Choi and Yun (2006) investigate the performance of several functions to calculate a threshold limit on the acceptable cost of minimum repair. Their model replaces the failed product if the expected cost of minimum repair exceeds the predetermined threshold. Iskandar and Murthy (2003), Iskandar et al. (2005), Chukova and Johnston (2006) and Chukova et al. (2006) apply the combination type imperfect repair models in the context of two-dimensional warranties. In these four

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papers, warranty region is divided in various ways into disjoint sub-regions with a priory decision on whether to pursue minimum or complete repair within each region. The objective is to determine the sub-regions so as to minimize the expected warranty cost.

An alternative approach is a generalization of the renewal process in which the product failure characteristics are revised after each failure as in the virtual age model proposed in Kijima (1989). In this model, the virtual age of the failed product is adjusted by a factor that reflects the degree of repair so as to bring it to a desired state somewhere between as good as new and as bad as old. Yanez et al. (2002) propose the use of Bayesian and maximum likelihood methods to estimate the model parameters for the generalized renewal process. Dagpunar (1997) and Dimitrov et al. (2004) use modified versions of the virtual age model.

Wang and Pham (1996a,b) and Bai and Pham (2005) use a further alternative and model the imperfect repairs in a single-dimensional warranty context as a quasi-renewal process. In the current paper, we extend their methodology to multi-dimensional warranties and adopt the appropriate version in both one and two-dimensional analyses. Due to the significance of the chronological age in warranty applications, quasi-renewal processes have greater intuitive appeal than the virtual age models in a warranty context. Quasi-renewal processes yield a mathematically convenient approach to calculate the number of failures within the warranty period.

The remainder of the paper is organized as follows. Section 2 presents a detailed description of the problem. Section 3 describes the methodology used to model the failure and repair process, defines a representative cost function, and develops different repair strategies. Renewal equations are also characterized in this section to calculate the expected number of failures under different types of two-dimensional warranties. Section 4 presents an application of the proposed approach in a real life industrial example. The approach is investigated under a variety of settings through computational experimentation in Section 5. Section 6 concludes the paper and offers some suggested directions for future research.

2. Problem description

The objective is to investigate the performance of alternative repair strategies in terms of the manufacturer's expected warranty cost under one- and two-dimensional warranties. The repair strategy has an effect on both the cost of a single repair and the number of repairs to be covered under warranty. Evidently, the total expected warranty cost is also a function of various other parameters such as product's reliability characteristics, the type of the warranty contract and the mathematical structure of the cost function. Before we introduce the detailed scheme within which we control these parameters and pursue our analyses, we make a few simplifying assumptions.

We first assume that buyers of a given product have similar usage patterns. Thus, the time until failure follows the same probability distribution. Next we assume that all claims made during the warranty period are valid and hence must be properly rectified by the manufacturer in accordance with the terms of the warranty contract. Finally, we consider the repair duration to be significantly smaller than the length of the warranty period so that repairs can be modeled to occur instantaneously.

With respect to repair actions, we study imperfect repairs based on a quasi-renewal process. The quasi-renewal process is characterized by a scaling parameter that alters the random variable corresponding to time until next failure after each renewal. In other words, this parameter indicates the degree of deterioration or improvement. For example, if the scaling parameter is between 0 and 1, it indicates deterioration; whereas if it is greater than 1, it indicates an improvement. Hereon, we refer to this parameter as the degree of repair. The degree of repair also determines the amount of change in the mean inter-failure time and the failure rate before and after the renewal.

To compare various policies, we use the expected total cost over the warranty period. Representative cost functions that address this issue for one- and two-dimensional warranties are proposed in Section 3.2.

3. Modeling the failure and repair process

In this part, we first present in Section 3.1, the multiple quasi-renewal processes to model the failure and associated repair process. Then in Section 3.2, representative cost functions for one- and two-dimensional warranties are introduced. In Section 3.3, different repair strategies are proposed. Lastly, calculation of the expected number of failures under one- and two-dimensional warranties is discussed in Section 3.4.

3.1. Multiple quasi-renewal process

In this section, the univariate quasi-renewal processes proposed by Wang and Pham (1996b) are generalized to multivariate distributions to model n-dimensional warranties. For a failure process defined along n-dimensions, let $\mathbf{X}_i = (X_{1i}, X_{2i}, \ldots, X_{ni})$, $i = 1,2,3,\ldots$ represent an n-dimensional random vector where X_{ki} denotes the length of the interval between the $(i-1)$th and ith successive renewals on the kth dimension with $X_{k0} = 0$ for $k = 1,2,\ldots,n$. Consider a counting process $\{N(x_1,x_2,\ldots,x_n);\ x_k > 0,\ k = 1,\ldots,n\}$ that represents the number of events in region $(0,0,\ldots,0)\times(x_1,x_2,\ldots,x_n)$. This process is an n-dimensional quasi-renewal process if

$$\begin{bmatrix} X_{1i} \\ \vdots \\ X_{ni} \end{bmatrix} = \begin{bmatrix} \alpha_1^{i-1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \alpha_n^{i-1} \end{bmatrix} \begin{bmatrix} Y_{1i} \\ \vdots \\ Y_{ni} \end{bmatrix} = \begin{bmatrix} \alpha_1^{i-1} Y_{1i} \\ \vdots \\ \alpha_n^{i-1} Y_{ni} \end{bmatrix}$$

where α_k is a positive real constant that measures the degree of repair in the kth dimension for $k = 1,2,\ldots,n$ and $\mathbf{Y}_i$ is an n-dimensional i.i.d. random vector for all i.

Let $F(y_{1i},y_{2i},\ldots,y_{ni})$ and $f(y_{1i},y_{2i},\ldots,y_{ni})$ be the c.d.f. and p.d.f. of $\mathbf{Y}_i=(Y_{1i},Y_{2i},\ldots,Y_{ni})$ for $i = 1,2,3,\ldots$ respectively. Then the cumulative distribution and density functions of $\mathbf{X}_i$ can be written as follows:

$$F_i(x_{1i},\ldots,x_{ni}) = F(\alpha_1^{1-i}x_{1i},\ldots,\alpha_n^{1-i}x_{ni}),$$
$$f_i(x_{1i},\ldots,x_{ni}) = \frac{\partial^n F_i(x_{1i},\ldots,x_{ni})}{\partial x_{1i}\ldots\partial x_{ni}} = \prod_{k=1}^{n} \alpha_k^{1-i} f(\alpha_1^{1-i}x_{1i},\ldots,\alpha_n^{1-i}x_{ni}).$$

The probability function of $N(x_1,x_2,\ldots,x_n)$ can be derived by using the relationship $N(x_1,x_2,\ldots,x_n) \geqslant i \Leftrightarrow S_i \leqslant (x_1,x_2,\ldots,x_n)$, where S_i is the occurrence point of the ith event. The probability that there will be i events within region $(0,0,\ldots,0) \times (x_1,x_2,\ldots,x_n)$ is

$$P(N(x_1,\ldots,x_n) = i) = P(S_i \leqslant (x_1,\ldots,x_n)) - P(S_{i+1} \leqslant (x_1,\ldots,x_n)),$$
$$P(N(x_1,\ldots,x_n) = i) = F^{(i)}(x_1,\ldots,x_n) - F^{(i+1)}(x_1,\ldots,x_n) i = 1,2,\ldots$$

where $F^{(i)}$ is the i-fold convolution of F with $F^{(0)}(x_1,x_2,\ldots,x_n) = 1$.

Consequently, the renewal function for the n-dimensional quasi-renewal process is obtained as follows:

$$M_q^n(x_1,\ldots,x_n) = E[N(x_1,\ldots,x_n)] = \sum_{k=0}^{\infty} kP(N(x_1,\ldots,x_n) = k)$$
$$= \sum_{k=1}^{\infty} F^{(k)}(x_1,\ldots,x_n)$$