Available online at www.sciencedirect.com
SCIENCE DIRECT®
Computers and Chemical Engineering 29 (2005) 1801–1804
Computers & Chemical Engineering
www.elsevier.com/locate/compchemeng

Explicit internal signal stochastic resonance and mechanic of noise-resistance in a chemical oscillator

Aizhong Lei, WeiGuo Xu, DaiPing Hu *, HuiZhang Shen

Institute of System Engineering, Shanghai Jiao Tong University, Shanghai 200052, China

Received 7 March 2004; received in revised form 1 March 2005; accepted 1 March 2005

Abstract

The explicit internal signal stochastic resonance (EISSR) that happened in the Willamowski–Rössler reaction model was investigated. The *z*-variable's output signal-to-noise ratio (SNR) as a function of the noise intensity shows non-monotonic behavior, indicating the occurrence of EISSR. It is first found that EISSR happened in the periodic-2 oscillations state. A worthy noting point is that the fundamental frequency of the intrinsic period signal of the system is shifted little with the change of noise intensity. EISSR is a cooperative effect of the intrinsic period signal and external noise or fluctuations. Our studies have shown that EISSR can lead to noise-resistance of the chemical oscillator.

Keywords: Stochastic resonance; Explicit internal signal stochastic resonance; The Willamowski–Rössler model; Signal-to-noise ratio

1. Introduction

The influence of noise on non-linear system has been extensively studied over the past decade. Random noise is intuitively considered a nuisance. It destroys signal detection and transduction. However, it has been found in many non-linear systems that noise can play a constructive role through stochastic resonance (SR) (Bezrukov & Vodyanoy, 1997; Gammaitoni, Hänggi, Jung, & Marchesoni, 1998; Wiesenfeld & Moss, 1995).

Stochastic resonance is a phenomenon wherein the response of a non-linear system to a weak periodic signal is optimized by the assistance of a particular non-zero level of noise. SR was first put forward by Benzi, Sutera, and Vulpiani (1981) to account for the periodic oscillations of the Earth's ice ages. It has taken place in a wide range of physical, chemical and biological systems (Matyjaśkiewicz, Krawiecki, Holyst, Kacperski, & Ebeling, 2001; McNamara, Wiesenfeld, & Roy, 1988; Vilar, Gomila, & Rubí, 1998). SR in some chemical reactions have been investigated by Schneider's group (Foster, Merget, & Schneider, 1996; Guderian, Dechert, Zeyer, & Schneider, 1996; Hohmann, Müller, & Schneider, 1996), Xin's group (Zhong & Xin, 2001, 2000a, 2000b; Zhong & Xin, 2000a,b) and our group (Li & Lei, 2003; Li & Zhu, 2001; Zhu & Li, 2002; Zhu, Li, & Liu, 2002). Recently, it is reported that the external signal can be replaced by internal signal, such as periodic oscillations in a deterministic system. When the system is perturbed by additive noise or multiplicative noise, the signal-to-noise ratio goes through a maximum with increment of noise intensity. In this letter, we study the effect of additive noise on a chemical oscillator. Our numerical results have shown that explicit internal signal stochastic resonance (EISSR) can lead to noise-resistance in a chemical oscillator and the frequency of the intrinsic period signal of the chemical oscillator is shifted little with the change of noise intensity.

1.1. Model

The model used in this letter is the Willamowski–Rössler model (Kapral & Showalter, 1995; Willamowski & Rössler, 1980). The non-dimensionalized chemical dynamical evolution equations of this system are given as

* Corresponding author. Tel.: +86 10 68912665; fax: +86 21 64072960.
E-mail addresses: leiazh@hotmail.com (A. Lei), xuwg@sjtu.edu.cn (W. Xu).

0098-1354/$ – see front matter © 2005 Published by Elsevier Ltd.
doi:10.1016/j.compchemeng.2005.03.006

follows:

$$\frac{dx}{dt} = a_1x - k_{-1}x^2 - xy - xz$$
$$\frac{dy}{dt} = xy - a_5y \qquad (1)$$
$$\frac{dz}{dt} = a_4z - xz - k_{-5}z^2$$

where a_1, a_4, a_5, k_{-1} and k_{-5} are the positive parameters. We select k_{-1} as the control parameter for our study. The value of other parameters are chosen as: $a_1 = 30.0$, $a_4 = 16.5$, $a_5 = 10.0$ and $k_{-5} = 0.5$. Fig. 1 shows the bifurcation diagram of Willamowski–Rössler model. Various dynamic behaviors, such as period-1, period-2 and chaos, can be seen in the Fig. 1. The system bifurcates from period-1 to period-2 at $k_{-1} \approx 0.87$. The hopf bifurcation point at $k_{-1} \approx 1.3$ is not shown in the diagram.

The periodic-1 and periodic-2 oscillations were picked as the state perturbed by the noise that is added directly to the

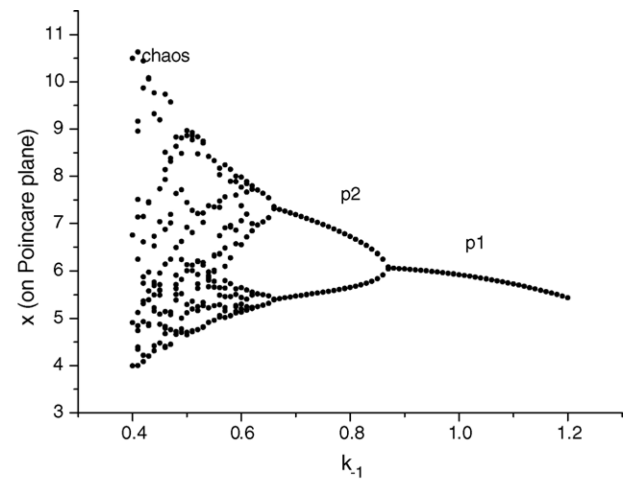

Fig. 1. Bifurcation diagram of the Willamowski–Rössler model: the dependence of the system's solution x on k_{-1}. Parameter values: $a_1 = 30.0$, $a_4 = 16.5$, $a_5 = 10.0$ and $k_{-5} = 0.5$. Notations used are: p1, period-1 oscillations; p2, period-2 oscillatory; chaos, a chaotic regime.

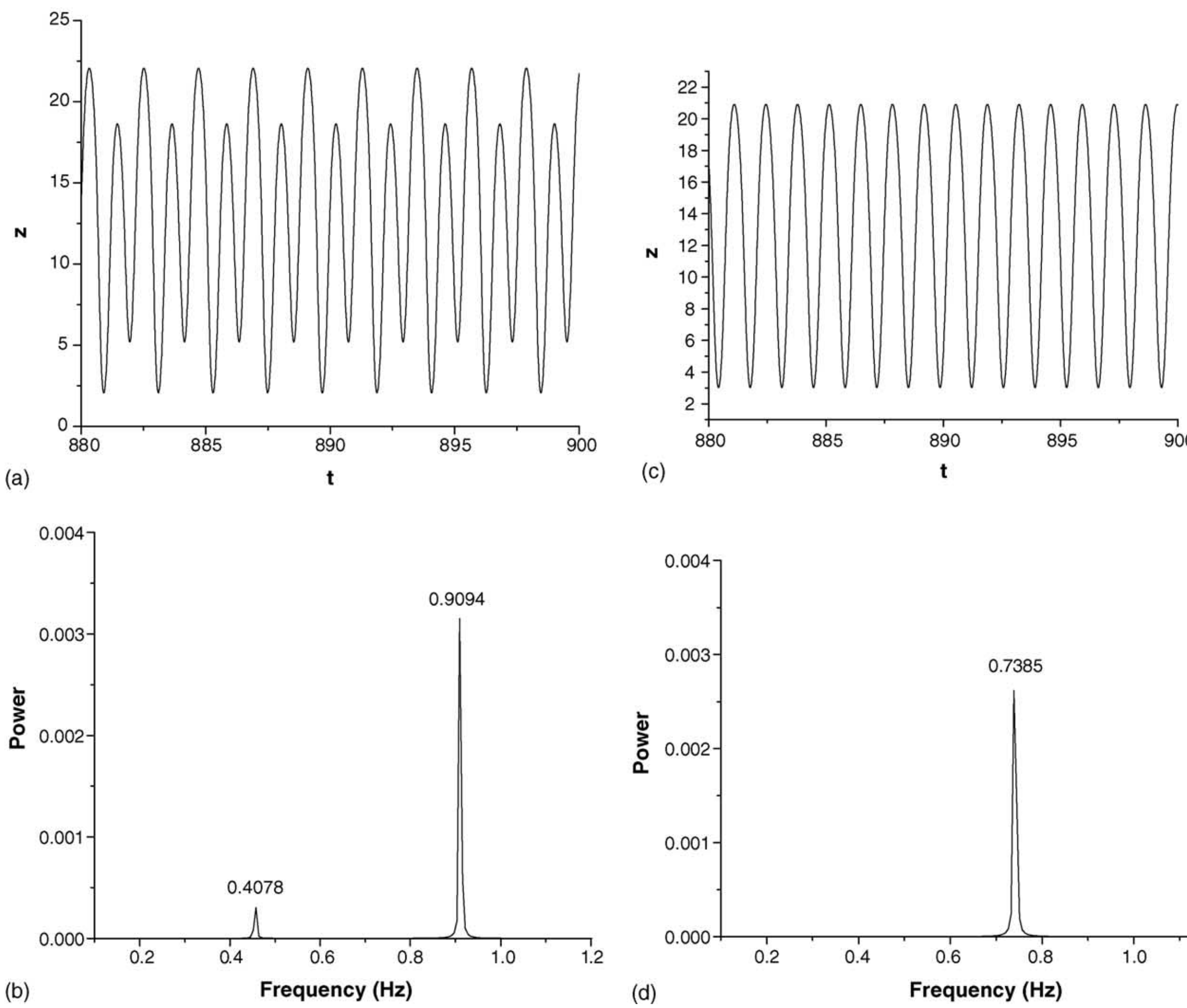

Fig. 2. (a) The variable z vs. time t, $k_{-1} = 0.7$; (b) the power spectrum acquired by fast Fourier transform of the times series of z when the parameter $k_{-1} = 0.7$; (c) the variable z vs. time t, $k_{-1} = 0.9$; (d) the power spectrum acquired by fast Fourier transform of the times series of z when the parameter $k_{-1} = 0.9$. When $k_{-1} = 0.9$, the system shows periodic-1 oscillation and the fundamental frequency is 0.7385 Hz. When $k_{-1} = 0.7$, the system shows periodic-2 oscillation and has two fundamental frequencies 0.9094 and 0.4078 Hz.