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On the use of stochastic resonance in sine detection

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Abstract

This paper deals with the use of stochastic resonance (SR) for detection purposes. The nonlinear physical phenomenon of SR generally occurs in dynamical bistable systems excited by a noisy sine: such systems are able to force cooperation between sine and noise such that the noise amplifies the sine. Because of this non-intuitive effect, the use of SR can be envisaged to detect small amplitude sines corrupted by additive noise. In this paper we recall some basics of detection and then show *why* SR can be used in sine detection context. After recalling some basics of SR in discrete time, we show *how* to use SR in a detection scheme. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Local signal-to-noise ratio; Markovian process (and operator); Nonlinear bistable models; Receiver operator characteristics; Sine detection; Stochastic resonance

1. Introduction

The stochastic resonance (SR) phenomenon has been the subject of considerable study in the last two decades (particularly see [14,33] and ref. therein). SR began with the study of climatic dynamics. In 1981, Benzi and co-workers [2,3] invoked this phenomenon to explain the Earth's climate change: the eccentricity of the Earth's orbit varies periodically in time, and if the amplitude of this variation is too small to explain the succession of ice ages and relatively warm periods, the periodical phenomenon is amplified by some perturbations. This cooperative effect between the coherent "signal" and the "noise" was called stochastic resonance (SR).

SR generally occurs in bistable dynamical systems attacked by a small periodic signal corrupted by noise: through the (nonlinear) internal dynamics of the system, the small signal is amplified by the presence of noise. To understand the idea, consider a particle moving in the bimodal potential pictured in Fig. 1. If the particle is excited by a small sinusoidal force and a small noise, it will oscillate within one of the two wells, and if the noise is too powerful, the particle will hop "completely randomly". Between these two extremes, for an optimal quantity of noise, the particle will hop between the two wells more or less at the frequency of the sine. The interesting point here is that the local output signal-to-noise ratio (SNR) plotted against the input noise variance presents a maximum.

Numerous studies have been developed to explain SR in continuous time using tools of statistical physics (master equation [30], Fokker–Planck equation [14,16,17], linear response theory [10,11,34]). Analogical simulations in electronic systems were also carried out [19,37]. A common assumption required by these theories is the Gaussianity of the noise. In order to use stochastic resonance (SR) in

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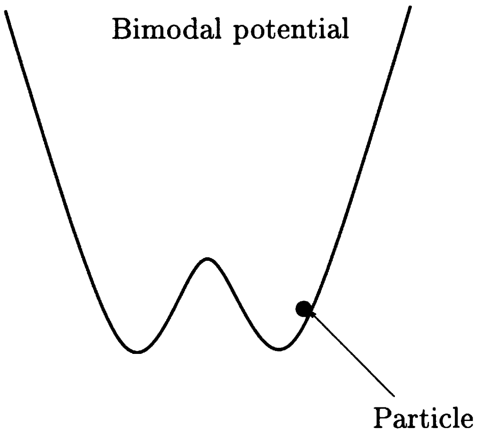

Fig. 1. Example of bimodal potential.

digital signal processing and due to the difficulty involved in the discretization of stochastic differential equations [24], we proposed in [40,39] a theory of SR in a class of dynamical discrete time systems. We showed that nonlinear bistable systems, autoregressive of order 1 (AR(1)), are able to create SR in both Gaussian and non-Gaussian contexts.

The existence of a maximum for the local output SNR against the power of the input noise has motivated several researchers to study the possibility of using SR in signal processing [1,5,12,21,22], particularly for detection purposes [6,13,20]. These attempts were generally unsuccessful. The main reason for these failures is that the studies deal with Gaussian noise (assumption required by the continuous-time theories of SR): there is no hope to improve the likelihood ratio test (LRT) [38]. In this paper, we will show that using SR in signal detection is possible, since the noise encountered is not Gaussian.

Section 2 will be a brief overview of the classical Neyman–Pearson strategy of detection. We will recall the detector obtained in the white Gaussian case. Then we will recall the performances of this detector in white Gaussian contexts and in some non-white and non-Gaussian contexts. Through this overview, we will see that the key parameter for the performance of the obtained detector is the local SNR.

In Section 3 we will recall the main results of SR in discrete time, and we will give some illustrations in several contexts. We will particularly show that SR can improve the local SNR and thus can be used in sine detection contexts.

Section 4 will make the link between the two previous sections: it will be devoted to the use of SR for detection purposes. Some comparisons with locally optimum detectors (LOD) will be proposed.

Finally, we will discuss some further investigations, and we will give concluding remarks in Section 5.

2. Brief overview of the Neyman–Pearson detection strategy

In this section, we will recall the basics of detection that lead us to use SR to improve the detection of a small noisy sine. The problem is the following: a signal r_n, $n = 0, \dots, N-1$ is observed. One of two hypotheses must be chosen: H_0 where there is noise only, and H_1 where the sine is present.

$$\begin{aligned} H_0 : r_n &= b_n, \quad n = 0, \dots, N-1, \\ H_1 : r_n &= b_n + \varepsilon_n, \quad n = 0, \dots, N-1, \end{aligned} \tag{1}$$

where b_n is the noise of probability density function (pdf) denoted as f_b and where $\varepsilon_n = \varepsilon \cos(2\pi n \lambda_0 + \varphi_0)$ is the sine. In the following, φ_0 is considered as a random variable, uniformly distributed on $[0; 2\pi]$ and ε is assumed to be small compared to the noise standard-deviation σ_b.

The classical Neyman–Pearson strategy consists in maximizing the detection probability $P_\mathrm{d} = \Pr[D_1|_{H_1}]$ for a fixed false alarm probability $P_\mathrm{fa} = \Pr[D_1|_{H_0}]$ where D_i means that hypothesis H_i has been chosen. This maximization problem leads to the test

$$\Lambda(\boldsymbol{r}) = \frac{f\boldsymbol{r}|_{H_1}(\boldsymbol{r})}{f\boldsymbol{r}|_{H_0}(\boldsymbol{r})} \underset{D_0}{\overset{D_1}{\gtrless}} \kappa, \tag{2}$$

where $\boldsymbol{r}$ denotes the vector of the observation data r_n and where $\Lambda(\boldsymbol{r})$ is the likelihood ratio ($f_{\boldsymbol{r}}|_{H_i}$ is the pdf of vector $\boldsymbol{r}$ under hypothesis H_i). Eq. (2) means that if $\Lambda(\boldsymbol{r})$ is higher than threshold κ, hypotheses H_1 is chosen, and H_0 otherwise. In the Neyman–Pearson strategy, the threshold is set to have a fixed false alarm probability $P_\mathrm{fa} = \int_\kappa^{+\infty} f_{\Lambda|_{H_0}}(x)\,\mathrm{d}x$ where $f_{\Lambda|_{H_0}}$ is the pdf of the likelihood ratio under hypothesis H_0. Eq. (2) is known as the LRT.

In the general case, this test is difficult to exploit, particularly when the pdf of the noise is bounded