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Signal Processing 83 (2003) 1193–1212

SIGNAL PROCESSING

www.elsevier.com/locate/sigpro

Design of detectors based on stochastic resonance

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Received 24 January 2002; received in revised form 28 August 2002

Abstract

This paper presents a study of the phenomenon of stochastic resonance in quantizers, and discusses the use of this phenomenon for the detection of weak sinusoidal signals in noise. Stochastic resonance in 2-level, symmetric 3-level, and symmetric multilevel quantizers is investigated. Expressions are derived for the signal-to-noise ratio (SNR) gain of the quantizers driven by a small amplitude sinsuoidal signal and i.i.d. noise. The gain depends on the probability density function (PDF) of the input noise, and for a given noise PDF, the gain can be maximized by a proper choice of the quantizer thresholds. The maximum gain G_{SR} is less than unity if the input noise is Gaussian, but several non-Gaussian noise PDFs yield values of G_{SR} exceeding unity. Thus, the quantizers provide an effective enhancement in the SNR, which can be utilized to design a nonlinear signal detector whose performance is better than that of the matched filter. The nonlinear detector in consideration consists of a stochastically resonating (SR) quantizer followed by a correlator. An asymptotic expression for the probability of detection of the SR detector is derived. It is shown that the detection performance of the SR detector is better than that of the matched filter for a large class of noise distributions belonging to the generalized Gaussian and the mixture-of-Gaussian families.

Keywords: Stochastic resonance; Threshold nonlinearity; Quantizer; Suboptimal detector; Passive detection; Non-Gaussian noise; Marine noise

1. Introduction

Detection of sinusoidal signals at low signal-to-noise ratios (SNRs) is a problem of great interest in the context of passive detection of targets in the ocean. The acoustic radiation from most targets contain distinct line components in the spectral domain, and are weak compared to the ambient noise. It is known [11] that a quadrature or incoherent matched filter is the optimal detector if a signal with unknown amplitude/phase is buried in Gaussian noise. Here, optimality refers to maximization of the probability of detection constrained by a fixed probability of false alarm. The matched filter though easy to implement and analyze is not optimal under conditions of non-Gaussian noise prevalent in marine environments. Optimal detectors in non-Gaussian noise are nonlinear and are not easy to implement. Hence, suboptimal nonlinear detectors which are easier to implement are often employed for the detection of signals in non-Gaussian noise. The aim of the present study is to design easily implementable nonlinear detectors based on stochastic resonance, and to analyze the performance of these detectors.

Stochastic resonance (SR) is the phenomenon of enhancement of signal transmission by certain

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0165-1684/03/$ - see front matter
doi:10.1016/S0165-1684(03)00039-2

Nomenclature

σ	noise standard deviation
γ_1	threshold
γ	normalized threshold
$x[n]$	input sequence
$y[n]$	output sequence
$s[n]$	incoming sinusoidal signal
$w[n]$	white noise sequence at the input
A_1	amplitude of incoming sinusoid
ϕ	phase of incoming sinusoid
Y_1	first Fourier coefficient of the sequence $E(y[n])$
$\sigma_y^2[n]$	variance of $y[n]$
$\overline{\sigma_y^2}$	average of the sequence $E(y[n])$
G	SNR gain of the system considered
$R_{\mathrm{in,out}}$	input, output SNR
p	order of generalized Gaussian
a, b	constants associated with generalized Gaussians
α	mixing parameter of mixture-of-Gaussians
β	ratio of standard deviations of mixture-of-Gaussians
c	constant associated with mixture-of-Gaussians
$R(.)$	ratio of component Gaussians for mixture-of-Gaussians
$P(.)$	probability of an event
$f(.)$	PDF of input noise
$F(.)$	probability distribution function of input noise
$C(.)$	the characteristic function for detector
$\tilde{C}(.)$	the normalized characteristic function for detector
$\Gamma(.)$	the Gamma function
$\Gamma(.,.)$	the incomplete Gamma function
$H(.,.)$	the continued fraction associated with the incomplete Gamma function
$T(.)$	test statistic for detector
η	detector threshold
$\chi_2'^2(.)$	noncentral chi-squared distribution
λ	noncentrality parameter of the chi-squared distribution
$Q_{\chi_2'^2(\lambda)}(.)$	the right-tail probability of the noncentral chi-squared PDF

nonlinear systems resulting from the addition of noise to the system (see [7,13] for recent reviews). Quantizers and other static nonlinear systems exhibit stochastic resonance [1–3,5,6,8,9,16,17]. For an SR system driven by a sinusoidal signal and stationary white noise, the output SNR increases as the input noise intensity σ^2 is increased over a certain range of values of σ^2. The probability of detection of a signal is maximized at the peak of the output SNR [10]. If the SNR gain is greater than unity, a combination of the SR system and a matched filter will yield, a detection performance that is better than that of the matched filter alone [4]. While conventional studies of stochastic resonance search for the optimal noise level, keeping the system fixed [8,10], in detection schemes the problem is reversed; the system parameters are optimized for a given noise level and type. This has been demonstrated by Chapeau–Blondeau for the detection of pulse trains using quantizers with optimizable thresholds [2].

In this paper also, we follow a similar approach. The organization of the paper is as follows. In Section 2, properties of stochastic resonance demonstrated by quantizers and their utility in signal detection are discussed qualitatively. It is shown that the maximum SNR gain is less than unity if the input noise is Gaussian, but the SNR gain can exceed unity if the noise is non-Gaussian. In Section 3, an algorithm for optimizing the SNR gain of the symmetric 3-level quantizer is developed, and is applied to generalized Gaussian and mixture-of-Gaussian noise probability density functions (PDFs). Section 4 presents a comparison between the optimal SNR gains of the symmetric 3-level, asymmetric 2-level and uniform symmetric multilevel quantizers. In Section 5, the monotonic dependence of the receiver operating characteristics on the SNR gain is established. It is also observed that if the noise PDF belongs to a subclass of the generalized Gaussian family, or a subclass of the Gaussian mixture family, the asymptotic performance of the quantizer-detector is significantly better than that of the matched filter but not as good as that of the optimal nonlinear detector. Section 6 contains the conclusions.