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Enhanced logical stochastic resonance under periodic forcing

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ABSTRACT

It was demonstrated recently that noise in an optimal window allows a bistable system to operate reliably as reconfigurable logic gates (Murali et al., 2009) [1], as well as a memory device (Kohar and Sinha, 2012) [11]. Namely, in a range of moderate noise, the system can operate flexibly, both as a NAND/AND/OR/NOR gate and a Set Reset latch. Here we demonstrate how the width of the optimal noise window can be increased by utilizing the constructive interplay of noise and periodic forcing, namely noise in conjunction with a periodic drive yields consistent logic outputs for all noise strengths below a certain threshold. Thus we establish that in scenarios where noise level is below the minimum threshold required for logical stochastic resonance (or stochastic resonance in general), we can add a periodic forcing to obtain the desired effects. Lastly, we also show how periodic forcing reduces the switching time, leading to faster operation of devices and lower latency effects.

1. Introduction

The phenomena of *logical stochastic resonance* (LSR) was demonstrated recently [1–10]: namely, when a bistable system is driven by two inputs it consistently yields a response mirroring a logic function of the two inputs in an optimal window of moderate noise. The same LSR elements can also be morphed into memory devices [11]. Further, the functionality of the LSR elements can be changed simply by varying an asymmetrizing bias. Thus it is clear that if noise is within some optimal window then it can play a constructive role in computational devices.

Unfortunately, the noise in a system does not always stay at the same level. Since LSR works in an optimal range, it is possible that the noise in the system is not sufficient to drive the system, i.e. the noise is below the minimum threshold of the optimal noise window. For instance, in case of thermal noise fluctuations in the environment such as ambient temperature, or certain internal processes, such as the work load of the device, may change the level of noise present in the system. So under very weak load or in very cold environments the system may not operate robustly.

Recently, Gupta et al. [12] showed that dynamical behavior equivalent to LSR can be obtained in a *noise-free* bistable system, subjected only to periodic forcing, such as sinusoidal driving or rectangular pulse trains. This opens up the possibility of studying the behavior of bistable elements subject to a periodic signal, as well as noise. In this paper, we will demonstrate how periodic forcing and noise interact constructively, thereby allowing us to obtain consistent logic and memory operations over a *much larger noise window*. Thus, by adding a periodic signal to a noisy nonlinear system we can obtain LSR consistently even if noise level is lower than the minimum threshold required to obtain LSR. Further, we can use two coupled bistable systems in which the output of one bistable system controls the amplitude of the periodic forcing fed into the other system. This suggests a way in which to *adaptively adjust the strength of periodic forcing depending on the noise level present in the system.*

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2. General principle

Consider the general nonlinear dynamical system,

$$\dot{x} = F(x) + b + I + D\eta(t) + A\,f(\omega t) \tag{1}$$

where $F(x)$ is a generic non linear function obtained via the negative gradient of a potential with two distinct stable energy wells. I is the input signal which is the sum of two square pulses encoding the two logic inputs, b is bias to asymmetrize the two potential wells, $\eta(t)$ is an additive zero-mean Gaussian noise with unit variance and D is the amplitude (intensity) of noise. The functional form of the periodic forcing is f, with ω being the frequency and A being the amplitude of the forcing.

A logical input- output can be obtained by driving the system with two trains of aperiodic square pulses: $I = I_1 + I_2$, where I_1 and I_2 encode the two logic inputs. Logic output can be obtained from the state x by defining a threshold value x^*. If $x > x^*$, then the logic output is interpreted to be 1, and 0 otherwise.

3. Explicit example

We now explicitly demonstrate this phenomena in the system given by:

$$\dot{x} = a_1(x - a_2x^3) + b + I_1 + I_2 + A\sin(\omega t) + D\eta(t) \tag{2}$$

where D is the amplitude of noise, b the asymmetrizing bias and the functional form of periodic forcing is sinusoidal where A is amplitude of the sinusoidal forcing. The parameters a_1 and a_2 control the height of the potential barrier and the location of potential minima. In absence of other terms, the height of potential barrier is $a_1/4a_2$ and the wells are at $\pm\sqrt{1/a_2}$ as shown in Fig. 1. Here we have taken $a_1 = 4$, and $a_2 = 5$. This function $F(x)$ is reasonably insensitive to noise and its two stable states are close to the encoded values of inputs. This helps to *cascade the gates, and feed the output directly as input, without any scaling factors*.

The logic inputs are presented to the system with I_1 and I_2 switching levels in an uncorrelated aperiodic manner. The inputs being 0 or 1, produce 4 sets of binary inputs (I_1, I_2): $(0,0), (0,1), (1,0), (1,1)$. These four distinct input conditions gives rise to three distinct values of I. Without loss of generality, consider the inputs to take value 0.5 when the logic input is 1, and value -0.5 when the logic input is 0. Hence, the input signal I, generated is a 3-level aperiodic wave form.

We choose 0 as our output determination threshold. If $x > 0$, i.e., when the system is in the positive potential well, then the logic output is interpreted to be 1, and 0 otherwise. Thus the logic output toggles as the system switches wells.

4. Results

We simulated the system in Eq. (2) for various possible frequencies and amplitude of the sinusoidal forcing and at various noise strengths [14]. We used $b = -0.5$, so the system is biased to function as AND gate [13]. We know that by changing the bias, we can easily switch to another logic operation. In this case, when bias is changed from -0.5 to 0.5, we obtain the OR gate. When bias is reduced to zero, we get a memory device. This effect arises from change in the symmetry and depths of the potential wells due to changing b. For brevity, we will show the results only for the representative AND gate.

We observe that for low noise strengths the system does not give the correct logical response in absence of periodic forcing as expected. However, as we apply some periodic forcing the system, gives the desired response. Notice that this response is obtained through interplay of noise and periodic forcing, as in absence of any one of these the system does not yield the desired response. *Only when both are present simultaneously, do we get the requisite output*, as shown in Fig. 2.

So when the noise level is low, it is not sufficient to induce the desired switch from one well to the other. Similarly, for high frequency or low amplitude, the sinusoidal forcing cannot drive the required hopping. However, when both are present,

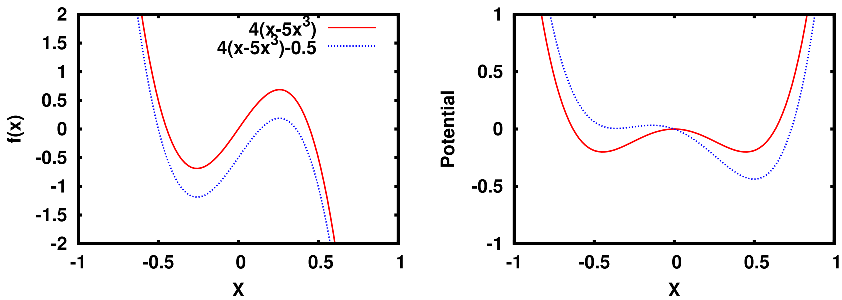

Fig. 1. For the system (2): (left) the function $F(x)$ and (right) the effective potential obtained by integrating the function $F(x)$, for different bias: (a) $b = 0$ (red solid) and (b) $b = -0.5$ (blue dashed line). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)