Commun Nonlinear Sci Numer Simulat 38 (2016) 257–266

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

Stochastic resonance in a piecewise nonlinear model driven by multiplicative non-Gaussian noise and additive white noise

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ARTICLE INFO

Article history:
Received 22 July 2015
Revised 22 January 2016
Accepted 28 February 2016
Available online 4 March 2016

Keywords:
Piecewise nonlinear model
Non-Gaussian noise
Stochastic resonance
Signal-to-noise ratio

ABSTRACT

The phenomenon of stochastic resonance (SR) in a piecewise nonlinear model driven by a periodic signal and correlated noises for the cases of a multiplicative non-Gaussian noise and an additive Gaussian white noise is investigated. Applying the path integral approach, the unified colored noise approximation and the two-state model theory, the analytical expression of the signal-to-noise ratio (SNR) is derived. It is found that conventional stochastic resonance exists in this system. From numerical computations we obtain that: (i) As a function of the non-Gaussian noise intensity, the SNR is increased when the non-Gaussian noise deviation parameter q is increased. (ii) As a function of the Gaussian noise intensity, the SNR is decreased when q is increased. This demonstrates that the effect of the non-Gaussian noise on SNR is different from that of the Gaussian noise in this system. Moreover, we further discuss the effect of the correlation time of the non-Gaussian noise, cross-correlation strength, the amplitude and frequency of the periodic signal on SR.

1. Introduction

The concept of stochastic resonance (SR) was presented by Benzi et al. [1] and Nicolis et al. [2] in 1981, and creatively applied to the study of the Quaternary glaciers issue. After that, as a non-linear phenomenon, SR has attracted considerable attention [3-13]. People's understanding of noises has changed and begun to notice the beneficial side of noise since the phenomenon of SR was put toward. In the study of SR and its related problems, noises are playing a positive and constructive role on the output of the system. At present, Gaussian noise has been successfully researched in aspects of theory and experiment. However, the research theory of non-Gaussian noise is not as mature as Gaussian noise. In fact, non-Gaussian noise can not be ignored, and it plays an irreplaceable role for the system [14,15] Some recent researches [16–21] show that the non-Gaussian noise often appears in physical systems, biological systems and neural systems. Fuentes et al. [17] studied SR in a bistable system driven by non-Gaussian noise. Wio et al. [18] analyzed a noise-induced transition when the system is driven by a noise source taken as colored and non-Gaussian. Wu et al. [19] investigated the steady-state probability density and SR of a stochastic system with coupling between non-Gaussian and Gaussian noise terms. Wu and Zhu [20] studied the phenomenon of stochastic resonance in a bistable system with time-delayed feedback driven by non-Gaussian noise. Zhang and Jin [21] investigated the mean first-passage time and SR in an asymmetric bistable system driven by multiplicative non-Gaussian noise and Gaussian white noise.

Above studies were performed mainly for continuous system driven by non-Gaussian noise. However, a large number of experimental results indicate that, for many experimental systems, the model is based on a segmentation system [22–27]

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http://dx.doi.org/10.1016/j.cnsns.2016.02.032
1007-5704/

Such as, controller, electronic circuit and superconducting device *etc.* Simpson et al. [22] analyzed the mixed-mode oscillation phenomenon of piecewise linear FHN model. Fiasconaro et al. [23] studied a class of activation resonance phenomenon of piecewise asymmetric system. Wang et al. [24] investigated SR of a bistable sawtooth system driven by correlated multiplicative and additive Gaussian white noises. Liang [25] propose a new parabolic bistable potential model with a additive Gaussian colored noise source, and studied the phenomenon of SR in this system. Jin and Li [26,27] investigated the mean first-passage time and SR in piecewise nonlinear system driven by multiplicative and additive Gaussian white noises with colored cross-correlation. But so far there has been little research on the theory of the effects of non-Gaussian noise on piecewise nonlinear system. And there has not yet been reported about SR in a piecewise nonlinear system driven by the non-Gaussian noise.

In this paper, we study the stochastic resonance phenomenon in a piecewise nonlinear system driven by the non-Gaussian noise. In Section 2, we obtain the FPK equation of system and the expression of steady-state probability density function by the path integral approach [17,28,29] and the unified colored noise approximation first [3,20,30–34]. Then the analytical expression of the SNR in a piecewise nonlinear system with a weak periodic signal is derived with two-state model theory. In Section 3, the SNR is presented and the influence of the non-Gaussian noise intensity, the non-Gaussian noise deviation parameter, the Gaussian noise intensity, the correlation time of the non-Gaussian noise, the cross-correlation strength, the amplitude and frequency of the periodic signal on SR is discussed. And the conclusions drawn are summarized in Section 4.

2. The SNR of a piecewise nonlinear system

We consider a piecewise nonlinear system driven by a periodic signal $A\cos(\Omega t)$ and two correlated noises, which is subjected to the following Langevin equation:

$$\dot{x}(t) = -U'(x) + A\cos(\Omega t) + x(t)\eta(t) + \xi(t). \tag{1}$$

Where A and Ω are the amplitude and frequency of the periodic signal, respectively. The deterministic part of the system (1) is a piecewise function, which corresponds to the potential function $U(x)$ [22–27]

$$U(x) = \begin{cases} \frac{a}{2}(x+1)^2 + \frac{k}{2}, & x < -\frac{\sqrt{3}}{3}, \\ \frac{b}{2}x^2, & |x| \le \frac{\sqrt{3}}{3}, \\ \frac{a}{2}(x-1)^2 + \frac{k}{2}, & x > \frac{\sqrt{3}}{3}, \end{cases} \tag{2}$$

and the parameter $a > 0,\ b < 0,\ k$ is constant, $U(x)$ has two stable states $x_{s1} = -1,\ x_{s2} = 1$ and an unstable state $x_u = 0$.

The potential function $U(x)$ image is shown in Fig. 1.

The parameters are chosen as follows: $(a = 0.01, b = -0.01, k = -0.0051)$.

In the system (1), the multiplicative noise$\eta(t)$ has a non-Gaussian distribution with [17,18,28]

$$\frac{d\eta(t)}{dt} = -\frac{1}{\tau}\frac{d}{d\eta}V_q(\eta) + \frac{1}{\tau}\varepsilon(t), \tag{3}$$

where,

$$V_q(\eta) = \frac{D}{\tau(q-1)} ln\left[1 + \frac{\tau(q-1)}{D}\frac{\eta^2}{2}\right], \tag{4a}$$

$$\langle \eta(t) \rangle = 0, \tag{4b}$$

$$\left\langle \eta^2(t) \right\rangle = \begin{cases} \frac{2D}{\tau(5-3q)}, & q \in \left(-\infty, \frac{5}{3}\right), \\ \infty, & q \in \left(\frac{5}{3}, 3\right). \end{cases} \tag{4c}$$

q is a measure of deviation of non-Gaussian noise $\eta(t)$ from a Gaussian distribution, τ is the correlation time of non-Gaussian noise. The variables $\varepsilon(t)$ and $\xi(t)$ are Gaussian white noises with the following statistical properties:

$$\langle \varepsilon(t) \rangle = \langle \xi(t) \rangle = 0, \tag{5a}$$

$$\left\langle \varepsilon(t)\,\varepsilon(t') \right\rangle = 2D\delta(t-t'), \tag{5b}$$

$$\left\langle \xi(t)\,\xi(t') \right\rangle = 2Q\delta(t-t'). \tag{5c}$$

$$\left\langle \varepsilon(t)\,\xi(t') \right\rangle = \left\langle \xi(t)\,\varepsilon(t') \right\rangle = 2\lambda\sqrt{DQ}\delta(t-t'). \tag{5d}$$