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Stochastic resonance in coupled underdamped bistable systems driven by symmetric trichotomous noises

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ABSTRACT

In this work, we analyze the phenomenon of stochastic resonance (SR) in coupled underdamped bistable systems induced by independent symmetric trichotomous noises. Firstly, the system is found to exhibit intermittent jumping motion between two wells in an optimal range of noise values. Then, the effects of noise on the output power spectrum of the system are characterized. It demonstrates that stochastic resonance behavior can occur above a particular value of noise strength. Finally, the signal-noise-ratio (SNR) for different parameters is computed numerically. Remarkably, for different parameters, a non-monotonic behavior of SNR is shown and the impacts of trichotomous noise and Gaussian white noise on the SNR curve are researched.

1. Introduction

Noise usually plays a disruptive role in nature. However, stochastic resonance (SR) where noise helps to make the system behave in a more coherent manner is one of the most interesting and appealing examples of the constructive role of noise in nature [1,2].

The original work on SR was mentioned by Benzi and coworkers for explaining the periodic recurrences of the Earth's ice age [3]. The first experimental observation of SR was performed while investigating the noise dependence of the spectral line of an ac-driven Schmitt–Trigger [4]. Then Gammaitoni et al. [5] suggested SR to be an example that illustrates the understanding of the cooperative behavior between noise and non-linear dynamics. SR is characterized by a noise-induced large response to a weak periodic signal. Thus, the SR effect can be used to amplify a weak signal by the injection of random noise [6]. The spectral power [7,8] and the signal-to-noise ratio (SNR) [9,10] mostly serve as measures for SR. Both measures show the non-monotonous behavior as functions of the noise intensity. In the past years, SR has been widely studied in a variety of fields, both in theory and in application [11–14].

Most of the above work for stochastic resonance phenomenon has been focused on systems subject to Gaussian noise [15–17] or dichotomous noise [18,19]. But, practically, there are a lot of noises that are not Gaussian or dichotomous. Trichotomous noise is a kind of three-level Markovian noise characterized by three parameters: amplitude, correlation time and flatness. Both trichotomous noise and dichotomous noise are stationary telegraph processes. However, trichotomous noise is a better representation of real noise than dichotomous noise in some cases. The trichotomous noise can approach to dichotomous noise under well-defined limiting procedures [20]. On the other hand, the flatness parameter φ of trichotomous noise can range from 1 to ∞, contrary to cases of the Gaussian colored noise ($\varphi=3$) and symmetric dichotomous noise ($\varphi=1$) [21,22]. This extra degree of freedom is useful in modeling actual fluctuations, e.g., thermal transitions between three configurations or states [21]. Some properties of trichotomous noise have been studied. For instance, Mankin and Ainsaar have studied the trichotomous noise-induced transitions [21]. Then, Mankin and his colleagues explored the phenomenon of stochastic resonance in some linear systems driven by trichotomous noise [22–25].

Although trichotomous noise has been largely explored in various dynamical systems, little has been done for coupled stochastic systems. In this paper, we demonstrate the constructive role of trichotomous noise assisted by a weak signal in a coupled underdamped bistable system. The structure of the paper is as follows. Firstly, in Section 2, the generation of the symmetric trichotomous noise is depicted and obtained numerically. Secondly, Section 3 is devoted to the description of the model for two coupled forced bistable scillators. In Section 4, we calculate power spectrum and signal-to-noise ratio numerically. Finally, conclusions are drawn in Section 5.

2. Generation of trichotomous noise

This section is devoted to the generation of symmetric trichotomous noise [20–24]. This noise is a random stationary Markovian

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process that consists of jumps between three values: a, 0 and $-a$. The jumps follow in time according to a Poisson process, while the values occur with the stationary probabilities: $P_s(a)=P_s(-a)=q$, $P_s(0)=1-2q$, with $0<q<1/2$ [20]. The transition probabilities between the states $\pm a$ and 0 can be obtained as follows:

$$P(-a,t+\tau|a,t)=P(a,t+\tau|-a,t)=P(\pm a,t+\tau|0,t)=q(1-e^{-\nu\tau}) \tag{1}$$

$$P(0,t+\tau|a,t)=P(0,t+\tau|-a,t)=(1-2q)(1-e^{-\nu\tau}), \quad \tau>0,\ \nu>0 \tag{2}$$

where the switching rate ν is the reciprocal of the noise correlation time: $\nu=1/\tau_c$. The mean and the correlation functions of the symmetric trichotomous noise $\xi(t)$ satisfy the following conditions:

$$\langle\xi(t)\rangle=0, \quad \langle\xi(t)\xi(t')\rangle=2qa^2e^{-\nu|t-t'|} \tag{3}$$

The noise intensity D is defined as

$$D=2\int_0^{\infty}\langle\xi(t),\xi(t+\tau)\rangle d\tau=4qa^2/\nu \tag{4}$$

The trichotomous process is a particular case of the kangaroo process [26] with flatness parameter:

$$\varphi=\langle\xi^4(t)\rangle/\langle\xi^2(t)\rangle^2=1/2q \tag{5}$$

If $q\to0$, the flatness $\varphi\to\infty$ i.e. for trichotomous noise, the flatness parameter φ can range from 1 to ∞.

In Fig. 1(a–b), we generate the symmetric trichotomous noise as a function of time for different noise intensity numerically. The numerical algorithm of the generation of trichotomous noise is validated by applying the scheme to a prototypical model system that possesses analytical solutions [21]. It is shown that the residence time extends with the increasing of the noise intensity D. It is worth remarking that the trichotomous noise can converge to the dichotomic noise in the limit $q=1/2$ which is presented in Fig. 1(c). Furthermore, the symmetric dichotomic noise can approach to a Gaussian white noise in the limit $a\to\infty$ and $\tau_c\to0$. In this case, $2a^2\tau_c$ equals a constant D_0, which represents the strength of the resulting Gaussian white noise, see Refs. [27–29].

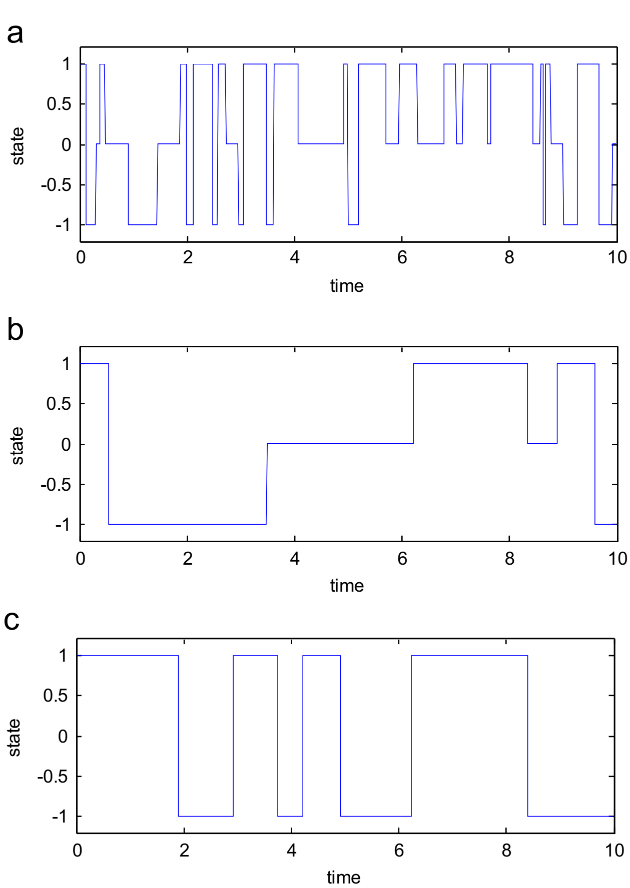

Fig. 1. The profiles of the symmetric trichotomous noise for the states 1, 0, −1. (a) $D=0.2$, $q=0.3$, (b) $D=1$, $q=0.3$, and (c) $D=1$, $q=1/2$.

3. Model system

Our system consists of two coupled underdamped bistable oscillators which are forced by two periodic signals and statistically independent noise sources. It can be described by the following dimensionless coupled stochastic differential equations [30]:

$$\ddot{x}=-r\dot{x}-\frac{dV_1(x)}{dx}+k(y-x)+\xi_1(x)+F_1(t) \tag{6}$$

$$\ddot{y}=-r\dot{y}-\frac{dV_2(x)}{dx}-k(y-x)+\xi_2(x)+F_2(t) \tag{7}$$

where r is the damping parameter and k is the coupling strength. The potentials of the two subsystems $V_i(x)$ for $i=1,2$ denote the reflection-symmetric quartic potentials:

$$V_i(x)=-a_ix^2/2+b_ix^4/4, \quad a_i>0,\ b_i>0 \tag{8}$$

The function $V_i(x)$ has two stable fixed points at $x=\pm\sqrt{a_i/b_i}$ and the top of the barrier is located at 0. The height of potential barrier is $\Delta V_i=a_i{}^2/4b_i$. In Fig. 2 the potential functions are displayed with $a_1=b_1=1$ and $a_2=1$, $b_2=1.5$, which leads to different activation barrier energies: $\Delta V_1=0.25$ and $\Delta V_2=1/6$, respectively. $\xi_1(t)$ and $\xi_2(t)$ are independent symmetric trichotomous noises which consist of the jumps between a, 0 and $-a$, defined as follows:

$$\langle\xi_i(t)\rangle=0 \tag{9}$$

$$\langle\xi_i(t)\xi_i(t')\rangle=2q_ia^2e^{-\nu_i|t-t'|} \tag{10}$$

$$\langle\xi_i(t)\xi_j(t')\rangle=0, \quad i\neq j \tag{11}$$

where $i,j=1,2$. The periodic driving signals are characterized by

$$F_i(t)=A_i\cos(\omega_it+\theta_i) \tag{12}$$

A_i is the amplitude, ω_i is the angular frequency of the periodic signal and θ_i is the initial phase with $i=1,2$. To allow for the adiabatic driving, we set the modulation frequency smaller than the relaxation one, say $\omega_1=\omega_2=\sqrt{2}/20$. Considering the subsystem x, Fig. 3 shows time series plot in the absence of the noise for various values of amplitude with $k=0$ and $r=0.25$. We can observe porototypical scenarios of no switching with $A_1=0.15$ shown in Fig. 3(a) and switching with $A_1=0.5$ shown in Fig. 3(b). In order to study stochastic resonance phenomenon, we consider a subthreshold signal amplitude $A_i<\Delta V_{1,2}$ that does not allow

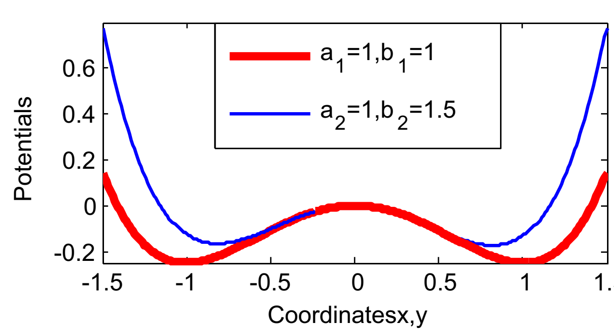

Fig. 2. The potential function in different cases: $a_1=b_1=1$ and $a_2=1$, $b_2=1.5$.