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Study on multi-frequency weak signal detection method based on stochastic resonance tuning by multi-scale noise

Peiming Shi [a,*], Xuejuan Ding [a], Dongying Han [b]

[a] College of Electrical Engineering, Yanshan University, Qinhuangdao, Hebei 066004, PR China
[b] College of Vehicles and Energy, Yanshan University, Qinhuangdao, Hebei 066004, PR China

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ABSTRACT

In practical engineering applications, useful information is often submerged in strong noise and the feature information is difficult to be extracted. Aimed at the detection problem of multi-frequency signal under colored noise background, a novel weak signal detection method based on stochastic resonance (SR) tuning by multi-scale noise is proposed. Firstly, noisy signal is processed by orthogonal wavelet transform to decompose the signal into multi-scale ingredients. According to the orthogonal wavelet transform coefficients characteristics of $1/f$ distribution, multi-scale noise is constructed so as to make the frequency-band containing the driving frequency be enhanced through SR system. Thus multi-frequency weak signal is detected. The method is effective to detect multi-frequency weak signal under colored noise background. Experiment signal analysis results show that the proposed method is simple for multi-frequency weak signal detection, and has good prospects for engineering applications.

1. Introduction

Stochastic resonance (SR) is a physical phenomenon in which some processing done on a signal can be improved when noise level is increased or specific noise is added to the system input [1–4]. Since the concept of SR was proposed by Benzi et al. in 1981 [1], SR has been widely used in the detection of weak signals [5–13]. Compared with the traditional de-noising method, SR utilizes noise instead of eliminating noise to improve signal-to-noise ratio to detect weak signals. The output signal of a nonlinear dynamic system can be enhanced by means of noise addition to the system based on the SR mechanism.

During the past decade, SR has been a hot research field of signal processing due to its advantages in weak signal detection [14–28]. The traditional SR method mainly focuses on small parameters including very low driving frequency ($f_0 \ll 1$ Hz), low amplitude ($A \ll 1$) and low noise intensity ($D \ll 1$), able to give an approximate analytical solution of SR Langevin equation. The detection of large frequency weak signals will have to in conjunction with other technical means to achieve [29,35,36].

There are many scholars have studied the detection and identification of single-frequency signal in strong noise. In practical engineering applications, the noise is generally non-Gaussian. Color noise study is closer to the actual situation [14,24]. Nozaki et al. studied the effects of colored noise with a $1/f^{\beta}$ power spectrum on the SR in sensory neurons. $1/f$ noise could be better than white noise for enhancing the response of an SR system to a weak signal [14]. Xu et al. have addressed realizing the SR by tuning system parameters with white noise and colored noise [19,20]. A new effect, named short-time SR, was explored in this nonlinear system for detecting frequency-shift keyed signals [29]. Duan and Abbott investigate bistable receiver response to binary modulated signals, versus the amount of noise [30], and propose a dynamical saturating system and numerically study its nonlinearity in terms of the output SNR and SNR gain [31]. A method of

* Corresponding author. Tel./fax: +86 335 7187169.
E-mail address: spm@ysu.edu.cn (P. Shi).

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weak-periodic-signal stochastic resonance in locally optimal processors with a fisher information metric is proposed [32]. He et al. realized the SR via multiscale noise tuning according to the property of $1/f$ noise [33,34]. Tan et al. realized SR with large parameters by the sub-sampling [35]. Leng et al. [36] developed methods of transforming a high frequency into a low frequency based on frequency re-scaling or frequency modulation to satisfy the requirements of traditional SR. These studies provide methods to the application of SR technology processing large parameters signals. However, there are a variety of frequency components in practical signals to be detected. Especially, for the detection problem of multi-frequency week noisy signal under the influence of strong noise, the extraction of useful information is very difficult, and it has an important practical significance. Therefore, for frequencies with a large difference, it needs further study.

In this paper, a novel method based on SR for multi-frequency weak signal detection is proposed in a color noise background. The main of this paper is organized as follows. Section 2 provides a brief introduction to the theory of SR and describes how to construct the common model of SR. Section 3 introduces the synthesis of multi-scale noise by the orthogonal wavelet coefficients characteristics of $1/f$ distribution. Section 4 describes the method for multi-frequency weak signal detection based on Re-scaling frequency-shifted band-pass SR (RFBSR) under colored noise background. Section 5 testifies the performance of the proposed method with the experimental data and presents the computational results for an experimental example. Finally, Section 6 provides the conclusion.

2. Theory of SR

The common model of SR is a bistable nonlinear system driven by periodic signal and white noise. It takes advantage of synergistic effect of the input signal and noise in nonlinear system, transferring noise energy to useful signal to resonate, thus achieving the purpose of identifying weak signals. The Langevin equation of bistable system is considered as follows

$$\frac{dx}{dt} = -U'(x) + s(t) + n(t) \tag{1}$$

where $s(t)$ is the input signal and $n(t)$ is the white noise. Let $E[n(t)] = 0E[n(t)n(t-\tau)] = 2D\sigma(\tau)$, where D is the noise intensity, $\sigma(\tau)$ is white noise with zero mean and unit variance, and $x(t)$ is the output signal. The potential function for the above bistable system can be denoted as

$$U(x) = -\frac{a}{2}x^2 + \frac{b}{4}x^4 \tag{2}$$

The above equation has two stable solutions and one unstable solution and its barrier height is $\Delta U = a^2/4b$ shown as Fig. 1.

When $s(t) = 0$, $x = \pm\sqrt{a/b}$, the potential energy is minimum and the system is most stable at the lowest point. When a weak periodic signal is input to the system, the signal energy cannot overcome the blocking of barrier ΔU and the output state moves only within a potential well. If the noise is input into the system, the noise energy will be partially transferred to the signal to produce interactions so as to overcome the system barrier. Thus the system makes transition between the two steady states. Due to the potential difference between the two steady states is much larger than the amplitude of the input signal so that output signal amplitude is greater than the input signal amplitude, which shows the SR phenomenon.

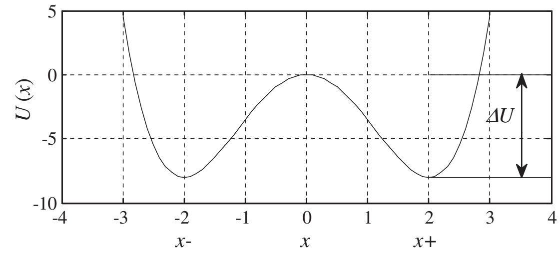

Fig. 1. The bistable potential function $U(x)$.

3. The synthesis of multi-scale noise

The white noise with a wide frequency range is decomposed by wavelet transform into multi-scale frequencies. Reconstruct the wavelet coefficients to synthesize a new distribution similar to the $1/f$. For band-limited noise through SR system can also generate Lorentz effect so that it can bring out SR by tuning the band-limited noise. $1/f$ process also known as pink noise is the most common noise in nature. Since a variety of practical engineering signals have the power spectrum of the $1/f$ type, $1/f$ process is widely used as a signal model. Its power spectral density is a power function inversely proportional to the frequency. The power spectral density of can be denoted as [37]

$$S_x(\omega) = \sigma_x^2/|\omega|^\gamma \tag{3}$$

where σ_x^2 is a constant (the variance of $1/f$ process), γ is the spectral parameter and $0 < \gamma < 2$, usually very close to 1.

Approximate $1/f$ process can be understood as only in a certain frequency range is $1/f$ distribution. The power spectral density value is between the interval $[k_1/|\omega|^\gamma, k_2/|\omega|^\gamma]$, wherein $0 \leqslant k_1 \leqslant k_2 < \infty$.

3.1. The orthogonal wavelet coefficients characteristics of 1/f distribution

Suppose $x(t) \in L^2(R)$ is a $1/f$ process and use wavelet bases $\psi(t)$ to transform $x(t)$ by discrete binary wavelet. The wavelet transform coefficient is

$$d_m(k) = 2^{m/2}\int_{-\infty}^{+\infty} x(t)\psi(2^m t - k)dt \tag{4}$$

The correlation value between any two wavelet transform coefficients can be expressed as

$$\begin{aligned} E[d_m(k)d_{m'}(k')] &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E[\psi_{m,k}(t)x(t)x(t')\psi_{m',k'}(t)]dtdt' \\ &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \psi_{m,k}(t)E[x(t)x(t')]\psi_{m',k'}(t)dtdt' \end{aligned} \tag{5}$$