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Effects of multiscale noise tuning on stochastic resonance for weak signal detection

Qingbo He *, Jun Wang

Department of Precision Machinery and Precision Instrumentation, University of Science and Technology of China, Hefei, Anhui 230026, PR China

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ABSTRACT

Noise enhanced signal detection via stochastic resonance (SR) is generally realized by white noise tuning with an optimal noise intensity. This paper explores a new mechanism of SR that is induced by the noise at multiple scales for enhanced detection of weak signals under heavy background noise. A strategy is proposed to realize the SR via multiscale noise tuning according to the property of $1/f$ noise. The presented new method combines the benefits of colored noise and parameter tuning to the SR phenomenon. Under the strategy, effects of noise intensity, analysis scale, and driving frequency on the SR are analyzed through numerical simulations. Three merits are displayed for the proposed multiscale noise-induced SR model: insensitivity to noise intensity, activity of multiple scale noise, and capability of detecting high frequency. A practical application to structural defect identification has confirmed the effectiveness of the proposed method in comparison with traditional methods.

1. Introduction

During the past three decades, stochastic resonance (SR) being one of the most exciting nonlinear phenomena has attracted considerable attentions in a wide range of research. Via the SR, the output signal of a nonlinear dynamic system can be enhanced with optimized signal-to-noise ratio (SNR) by means of noise addition to the system [1]. The phenomenon of SR benefits weak signal detection under the background noise. As compared to traditional techniques that mainly focus on how to suppress the noise, the SR has the merit of signal enhancement by the aid of the noise. Taking the SR as a mechanism for signal detection has, hence, been explored in various studies [2–19]. These studies indicate that noise enhanced signal detection is more powerful than noise suppression-based techniques especially when the target signal is corrupted by heavy noise.

The noise plays an active role in weak signal detection via SR. When the input noise intensity is low, the system output is not sufficiently driven to trace the input signal. For a high level of input noise, the system response is so strong that the output becomes random. Only when the input noise reaches an optimal level, the system response is capable of following the input signal so that the output SNR is maximized in a nonlinear mechanism. This leads to an intuitive belief that the optimal intensity of the noise should be adjusted in order to achieve the best performance of signal detection. Lots of past studies addressed the viewpoint of noise tuning when the noise is independent of the driving signal. However, there are three practical issues that may cause limitations of this approach on engineering applications. The first issue is that practical noise is usually not Gaussian [3,11]. Second, it might not be a suitable option of adding noise to or removing noise from the practical systems [7]. In addition, the third one is that practical input of a system may have a driving frequency being higher than 1 Hz [10,12,15]. For instance, these three issues can be found in the vibration signals measured from a defective rolling element bearing. Therefore, it is necessary to study how to overcome these limitations for achieving a general utility of noise enhanced signal detection.

Focusing on the above issues, relative works have been also reported. Nozaki et al. studied the effects of colored noise with a $1/f^{\beta}$ power spectrum on the SR in sensory neurons [3]. They showed that under certain circumstances, $1/f$ noise could be better than white noise for enhancing the response of an SR system to a weak signal. Recently, Xu et al. have addressed realizing the SR by tuning system parameters instead of tuning noise as the nature of the target signal changes [7]. This method was also used to analyze the system with colored noise [8]. Leng et al. [12] and Tan et al. [15] developed methods of transforming a high frequency into a low frequency based on frequency re-scaling or frequency modulation to satisfy the requirements of traditional SR. In summary, these three aspects have respective merit as below: colored noise may approach practical noise case and act as a better role in SR than the white noise; system parameter tuning can avoid the requirement of noise intensity tuning; high frequency detection may be realized via the SR through frequency transform techniques. However, until now there is not a method that can

* Corresponding author. Fax: +86 551 3607894.
E-mail address: qbhe@ustc.edu.cn (Q. He).

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unify these three merits, including colored noise, parameter tuning, and high frequency detection, to realize a general SR solution. Therefore, it will be a new attempt to combine these merits to make more general and more powerful detection of weak signals.

Motivated by the aforementioned analysis, this paper focuses on realizing the SR through considering a unified scheme by means of colored noise and parameter tuning. A novel principle, called multiscale noise tuning, is proposed to adjust the noise at multiple scales according to the $1/f$ noise property, instead of tuning the intensity of white noise. The multiscale noise tuning is realized by the discrete wavelet transform (DWT) to produce an approximate $1/f$-form distribution for the noise at multiple scales. In this $1/f$ distribution, the energy of multiscale noise may be transferred by the SR to the target frequency in a more effective way. The proposed SR with multiscale noise tuning can deal with high frequency detection as well as multiscale colored noise problems. This method is verified by means of numerical simulation and a practical application to structural defect identification.

The rest of the paper is arranged as follows. In Section 2, the bistable SR model is briefly introduced. In Section 3, the new strategy of multiscale noise tuning for SR is proposed. In Section 4, numerical simulations are conducted to evaluate the effects of the presented SR model. In Section 5, a practical case is provided to verify the value of the presented method in engineering applications. Finally, Section 6 draws concluding remarks.

2. Bistable SR model

To describe the SR, the overdamped motion of a Brownian particle in a bistable potential is considered in the presence of noise and periodic force as follows:

$$\frac{dx}{dt} = -U'(x) + A_0 \sin(2\pi f_0 t + \varphi) + n(t) \tag{1}$$

where A_0, f_0, and φ are amplitude, driving frequency, and initial phase of the periodic signal, respectively; and $n(t)$ is the noise. In Eq. (1), $U(x)$ denotes the reflection-symmetric quartic potential as below:

$$U(x) = -\frac{a}{2}x^2 + \frac{b}{4}x^4 \tag{2}$$

where a and b are barrier parameters with positive real values. Let $n(t) = \sqrt{2D}\xi(t)$ with $\langle n(t)n(t+\tau)\rangle = 2D\delta(t)$, in which D is the noise intensity and $\xi(t)$ represents a Gaussian white noise with zero mean and unit variance. Then Eq. (1) can be written as

$$\frac{dx}{dt} = ax - bx^3 + A_0 \sin(2\pi f_0 t + \varphi) + \sqrt{2D}\xi(t) \tag{3}$$

For the bistable model expressed in Eq. (3), the most important feature of the output amplitude is that it depends on the noise intensity D as shown as follows [2]:

$$\bar{x}(D) = \frac{A_0 \langle x^2 \rangle_0}{D} \frac{r_k}{\sqrt{r_k^2 + \pi^2 f_0^2}} \tag{4}$$

where r_k is the Kramers rate

$$r_k = \frac{1}{\sqrt{2}\pi} \exp\left(-\frac{\Delta U}{D}\right) \tag{5}$$

and $\langle x^2 \rangle_0$ is the D-dependent variance of the stationary unperturbed system ($A_0 = 0$). Eq. (4) means that the output amplitude first increases with rise in noise intensity, reaches a maximum, and then decreases again. This is the celebrated SR effect. Thus, via the SR phenomenon, the response of the system can be manipulated by changing the noise intensity.

To well understand the SR, the power spectral density $S(f)$ of the system response has been analyzed in [2]. In summary, the $S(f)$ contains two parts, $S_s(f)$ and $S_n(f)$, corresponding to contribution of the driving signal and the noise, respectively, as below:

$$S_s(f) = (\pi/2)\bar{x}^2\left[\delta(f - f_0) + \delta(f + f_0)\right] \tag{6}$$

$$S_n(f) = \left[1 - \frac{1}{2}\left(\frac{A_0 x_m}{D}\right)^2 \frac{r_k^2}{r_k^2 + \pi^2 f_0^2}\right] \frac{r_k x_m^2}{r_k^2 + \pi^2 f^2} \tag{7}$$

It can be seen that $S_n(f)$ is expressed as the product of the Lorentzian curve obtained with no input signal ($A_0 = 0$) and a factor that depends on the forcing amplitude A_0. The Lorentzian distribution is characterized by concentrating most of the noise energy into the low-frequency region. That is to say, white noise energy that distributes uniformly in the whole spectrum will mostly be accumulated into low frequencies by the nonlinear bistable system. The energy concentration then leads to the SR phenomenon for the low-frequency driving component.

The above theoretical analysis of the bistable SR model is under the assumption that the frequency and amplitude of a periodic signal as well as the noise intensity are all smaller than one [20]. This may be explained by the adiabatic approximation or linear response theory. To make the classical SR approach capable in detecting a high frequency, a parameter tuning approach can be derived based on normalized scale transformation as follows [10]. Mathematically, let $y = x\sqrt{b/a}$, and $\tau = at$. Then, Eq. (3) can be written as

$$a\sqrt{\frac{a}{b}}\frac{dy}{d\tau} = a\sqrt{\frac{a}{b}}y - a\sqrt{\frac{a}{b}}y^3 + A_0 \sin\left(\frac{2\pi f_0 \tau}{a} + \varphi\right) + \sqrt{2D}\xi\left(\frac{\tau}{a}\right) \tag{8}$$

Dividing Eq. (8) by the scaling factor $K = \sqrt{a^3/b}$ on both sides, we can get the following equation:

$$\frac{dy}{d\tau} = y - y^3 + \sqrt{\frac{b}{a^3}}\left[A_0 \sin\left(\frac{2\pi f_0 \tau}{a} + \varphi\right) + \sqrt{2D}\xi\left(\frac{\tau}{a}\right)\right] \tag{9}$$

Eq. (9) is the normalized form of Eq. (3). They can be considered to be the same one in nature. From these two equations, it can be seen that the frequency of the driving signal is normalized to be $1/a$ times of the original frequency in the new model. The frequencies of the noise are also normalized in the same form. Therefore, the model of Eq. (9) can be used for detection of weak signal with a high frequency. With the normalized scale transformation, choosing a large parameter a can normalize a high frequency (> 1 Hz) to be much smaller than one, which hence satisfies the requirement of the classical SR.

3. SR with multiscale noise tuning

The principle of SR has been used to detect weak signals embedded in heavy noise. By changing the intensity of noise, weak signal will be clearly highlighted from the noise. However, there also exists some trouble in white noise tuning in a measured signal, e.g., noise removal from the frequency band containing the driving frequency. When noise intensity cannot be conveniently adjusted, it is meaningful to study whether an optimized SR can be conducted through signal processing techniques. As demonstrated in Fig. 1, the Lorentzian distribution appears not only for the white noise but also for the band-limited noise, which indicates that the SR effect can be driven by different bands of noise. Therefore, it is possible to realize the SR by tuning the band-limited noise.