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Exploring weak-periodic-signal stochastic resonance in locally optimal processors with a Fisher information metric

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ABSTRACT

For processing a weak periodic signal in additive white noise, a locally optimal processor (LOP) achieves the maximal output signal-to-noise ratio (SNR). In general, such a LOP is precisely determined by the noise probability density and also by the noise level. It is shown that the output–input SNR gain of a LOP is given by the Fisher information of a standardized noise distribution. Based on this connection, we find that an arbitrarily large SNR gain, for a LOP, can be achieved ranging from the minimal value of unity upwards. For stochastic resonance, when considering adding extra noise to the original signal, we here demonstrate via the appropriate Fisher information inequality that the updated LOP fully matched to the new noise, is unable to improve the output SNR above its original value with no extra noise. This result generalizes a proof that existed previously only for Gaussian noise. Furthermore, in the situation of non-adjustable processors, for instance when the structure of the LOP as prescribed by the noise probability density is not fully adaptable to the noise level, we show general conditions where stochastic resonance can be recovered, manifested by the possibility of adding extra noise to enhance the output SNR.

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1. Introduction

Stochastic resonance (SR), originally introduced in the field of climate dynamics [1], is now emerging as a nonlinear signal processing method [2–8]. This method considers the possibility of adding an appropriate amount of noise to a nonlinear system (or network) in order to improve its performance described by an appropriate quantitative measure, such as the output signal-to-noise ratio (SNR) [2,3,6–14], the mutual information [15,16], the Fisher information [17–19], the detection probability [20–39], the mean-square-error of estimator [26,40], etc. A proven SR result is that, within the regime of validity of linear response theory, the output–input SNR gain cannot exceed unity for a nonlinear system subjected to a weak sinusoidal signal plus Gaussian white noise [4,8,41]. But, beyond the conditions where linear response theory applies, the possibility of SNR gain above unity is demonstrated for certain static nonlinearities [6,7] and dynamical systems [13,14]. More recently, many significant studies on SR in the areas of statistical signal detection and estimation [21–40] show the applicability of SR in nonlinear signal processing for the improvement of system performance by noise. However, most studies of SR first establish a fixed nonlinearity, and observe the noise-enhanced phenomenon therein. When an optimal processor can be updated according to the actual noise, specific examples [18,26,30,31] showed that the updated optimal processor operating on the data with extra noise can

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outperform the original optimal processor operating on the original data without extra noise. For the generic situation of the detection of a deterministic weak signal in noise, it is shown that the asymptotic efficacy of a locally optimal detector is generally determined by the Fisher information of the noise distribution [42]. Then, based on a Fisher information inequality, we prove that the improvement by adding noise is impossible for the detection probability of a weak known signal [42].

In this paper, we focus on the possibility of the SR effect in a locally optimal processor (LOP) that processes a weak periodic signal in additive white noise [24,44,45]. In this case, Zozor and Amblard [24] demonstrated that a LOP possesses the maximal output SNR. We further demonstrate that the output SNR of a LOP is closely related to the Fisher information of the noise probability density function (PDF) [43]. It is interesting to note that the output–input SNR gain of a LOP is given by the Fisher information of a standardized noise PDF. It is well known that a standardized Gaussian PDF has a minimum Fisher information of unity [44]. As a consequence, for any non-Gaussian noise, it is always possible to achieve an output–input SNR gain of a LOP larger than unity. Some types of noise and their corresponding LOPs are discussed for obtaining an arbitrary large output–input SNR gain. When adding extra noise to the original signal, we assume that the LOP can be updated according to the composite noise, aiming to achieve the maximal output SNR. Then, we prove that the updated LOP, via the Fisher information convolution inequality, is unable to improve the output SNR. This result generalizes a proof that existed previously only for a weak periodic signal in additive Gaussian noise [4,8,41]. This result and its domain of applicability leave open the possibility of SR or improvement by noise in situations with less flexibility, for instance, when the exact optimal updated LOP is not accessible or too complex to be implemented. Then, we further prove that, if the structure of a normalized LOP is a function of the noise root-mean-square amplitude, then such a prescribed LOP can exhibit the SR effect. Moreover, utilizing dichotomous noise as the added noise to the signal, a family of LOPs is elicited with their structures as a function of the root-mean-square amplitude of dichotomous noise. The SR effect is shown to always occur in such a prescribed LOP by increasing the added noise level to the special value given by the LOP. Based on the relationship of Fisher information of noise distribution and the output SNR, we show a new example of the Fisher information equality for the uniform noise and the dichotomous noise. Finally, some open questions are discussed.

2. No SR effect in an updated LOP

2.1. SNR gain of a LOP

Consider a static (memoryless) nonlinearity g with its output

$$y(t)=g[x(t)], \tag{1}$$

where $x(t)=s(t)+z(t)$ is a signal-plus-noise mixture input. The component $s(t)$ is a periodic signal with a maximal amplitude A $(0<|s(t)|\leq A)$ and period T. The zero-mean white noise $z(t)$, independent of $s(t)$, is with the PDF f_z and a root-mean-square amplitude σ_z [7]. The input SNR for $x(t)$ can be defined as the power contained in the spectral line $1/T$ divided by the power contained in the noise background in a small frequency bin ΔB around $1/T$ [7], that is

$$R_{\text{in}}=\frac{|\langle s(t)\exp[-i2\pi t/T]\rangle|^2}{\sigma_z^2\Delta t\Delta B}, \tag{2}$$

where Δt indicates the time resolution in a discrete-time implementation and the temporal average defined as $\langle\cdots\rangle=(1/T)\int_0^T\cdots dt$ [7]. Here, we assume $\Delta t\ll T$ and observe the output $y(t)$ for a sufficiently large time interval of NT $(N\gg 1)$. Then, the practical discrete-time white noise $z(j\Delta t)$ has the autocorrelation function $\mathrm{E}[z(j\Delta t)z(j\Delta t+k\Delta t)]=\sigma_z^2\Delta t\delta(k\Delta t)$ with the discrete-time version of the Dirac delta function $\delta(k\Delta t)=1/\Delta t$ for $k=0$ and zero otherwise [7]. Here, σ_z^2 is the variance of zero-mean white noise $z(t)$ [7]. Similarly, based on the cyclostationarity property of $y(t)$, the output SNR for $y(t)$ is given by

$$R_{\text{out}}=\frac{|\langle \mathrm{E}[y(t)]\exp[-i2\pi t/T]\rangle|^2}{\langle \mathrm{var}[y(t)]\rangle\Delta t\Delta B}, \tag{3}$$

with nonstationary expectation $\mathrm{E}[y(t)]$ and nonstationary variance $\mathrm{var}[y(t)]$ [7].

Assume $s(t)$ is weak $(A\to 0)$, and make a Taylor expansion of g around z at a fixed time t as

$$y(t)=g[z+s(t)]\approx g(z)+s(t)g'(z), \tag{4}$$

with $g'(z)=dg(z)/dz$ existing for almost all z. Here, the Taylor expansion of g is up to first order in the small signal $s(t)$. We further assume that g has zero mean and finite variance under f_z, i.e. $\mathrm{E}[g(z)]=\int_{-\infty}^{\infty}g(z)f_z(z)\,dz=0$ and $\mathrm{E}[g^2(z)]=\int_{-\infty}^{\infty}g^2(z)f_z(z)\,dz<\infty$. For an arbitrary memoryless nonlinearity g, the zero mean of $\mathrm{E}[g(z)]$ is not restrictive since any arbitrary g can always include a constant bias to cancel this average [44,45]. Therefore, we have

$$\mathrm{E}[y(t)]=\mathrm{E}[g(z)]+s(t)\mathrm{E}[g'(z)]\approx s(t)\mathrm{E}[g'(z)]. \tag{5}$$

Using Eqs. (4) and (5), we obtain

$$\begin{aligned}\mathrm{var}[y(t)]&=\mathrm{E}[y^2(t)]-\mathrm{E}[y(t)]^2\\&\approx \mathrm{E}[y^2(t)]-s^2(t)\mathrm{E}^2[g'(z)]\\&\approx \mathrm{E}[g^2(z)]+2s(t)\mathrm{E}[g(z)g'(z)]+s^2(t)\{\mathrm{E}[g'^2(z)]-\mathrm{E}^2[g'(z)]\}\\&\approx \mathrm{E}[g^2(z)]+2s(t)\mathrm{E}[g(z)g'(z)],\end{aligned} \tag{6}$$

up to first order in the small signal $s(t)$. Here, as $A\to 0$ $(0<|s(t)|\leq A)$, the higher-order term of $s^2(t)\{\mathrm{E}[g'^2(z)]-\mathrm{E}^2[g'(z)]\}$ is neglected [24,44,45]. Substituting Eqs. (5) and (6) into Eq. (3), we have

$$\begin{aligned}R_{\text{out}}&\approx\frac{|\langle s(t)\exp[-i2\pi t/T_s]\rangle|^2}{\Delta B\Delta t}\frac{\mathrm{E}^2[g'(z)]}{\langle \mathrm{E}[g^2(z)]+2s(t)\mathrm{E}[g(z)g'(z)]\rangle}\\&\approx R_{\text{in}}\sigma_z^2\frac{\mathrm{E}^2[g'(z)]}{\mathrm{E}[g^2(z)]},\end{aligned} \tag{7}$$

where the first-order term $2s(t)\mathrm{E}[g(z)g'(z)]$, compared with $\mathrm{E}[g^2(z)]$ and $\mathrm{E}^2[g'(z)]$, has no contribution for the calculation of R_{out} in the weak signal condition ($A\to 0$ and $0<|s(t)|\leq A$). The above derivations of Eqs. (5)–(7) are valid in the limit of a vanishing $s(t)$ [44,45].

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