Available online at www.sciencedirect.com
ScienceDirect
Mechanical Systems and Signal Processing 21 (2007) 2267–2279
Mechanical Systems and Signal Processing
www.elsevier.com/locate/jnlabr/ymssp

Engineering signal processing based on adaptive step-changed stochastic resonance

Li Qiang[a,*], Wang Taiyong[a], Leng Yonggang[a], Wang Wei[b], Wang Guofeng[a]

[a]School of Mechanical Engineering, Tianjin University, Tianjin 300072, PR China
[b]School of Management, Tianjin University, Tianjin 300072, PR China

Received 7 September 2006; accepted 4 October 2006
Available online 27 November 2006

Abstract

Weak signal detection which is under the condition of adiabatic elimination in large parameters can be solved by step-changed stochastic resonance (SCSR) presented by our group. Adaptive SCSR based on approximate entropy (*ApEn*) is also proposed in this paper, and it can get the best result of SCSR adaptively. Our analysis shows that the *ApEn* value of periodic signal is related to its frequency and signal-to-noise ratio (SNR), but not to the change of its amplitude and phase. So a periodic signal with definite SNR whose frequency is to be detected can be made under the same sampling condition as the raw data, and its *ApEn* is calculated as a standard reference. By adjusting the structural parameters and calculation step automatically, a series output of the bistable system can be got, and an *ApEn* distance matrix can be constructed. After getting the minimum value of the matrix, the best parameters of the non-linear system and calculation step can be obtained. Two examples of detecting weak signal mixed with heavy noise are presented in the end to illustrate that SCSR and its adaptive solution are effective for signal processing.

Keywords: Adaptive stochastic resonance; Approximate entropy; Bistable system; Rolling bearing; Fault diagnosis

1. Introduction

When noise is added to a system, the output of the system usually deteriorates in quality. However, in some systems, adding a proper amount of noise will enhance the signal-to-noise ratio (SNR) of the system output or response, rather than decrease it. This is currently the so-called stochastic resonance (SR) [1–6]. On the view of frequency domain, SR means that, when a sinusoidal driving force mixed with noise is inputted into a non-linear system, a maximum spectral spike at the driving frequency of the system response spectrum can be observed through varying the noise intensity. An SR phenomenon has been extensively paid attention to in such fields as weak signal detection [4], neural information coding [5], etc. It relates to a wide variety of physical systems [6,7], including monostable systems, multistable systems, and threshold systems.

*Corresponding author. Tel.: +86 22 27402774; fax: +86 22 27404536.
E-mail address: lqiww@yahoo.com.cn (L. Qiang).

0888-3270/$ - see front matter
doi:10.1016/j.ymssp.2006.10.003

At present, there has been a large amount of literature about the theoretical development and the applications of SR in bistable systems [4,8–11]. However, the majority of the theoretical studies in this area have been focusing on small parameter signals (the frequency and amplitude of a periodic signal and noise intensity are all smaller than one). This may be explained by the fact that most of the studies were restricted by adiabatic approximation and linear response theory, where these parameters were assumed to be small. Nevertheless, periodic driving with large parameters (frequency and/or amplitude and/or noise intensity can be much larger than one) in real world can be often encountered. For example, the frequencies of mechanical faults of rotating machinery are usually much higher than one. In this paper, the case of a high frequency and weak periodic signal hidden in heavy noise will be investigated. Here, the "weak" means small compared to the noise level. By the novel method of step-changed stochastic resonance (SCSR) [12–15] proposed by our group, an SR-like spectral spike at the frequency of the weak periodic signal can be obtained in the output spectrum of a dynamic bistable system.

At the same time, a new adaptive solution of the SCSR based on the approximate entropy (*ApEn*) is also presented. The advantage of the method is that it can detect a weak signal in the presence of strong noise or of the same frequency noise (which is the part with spectra close to the input frequency) by SCSR automatically. In our work, we concentrate on the numerical study of the behavior of the large parameter bistable SCSR and its adaptive solution. Numerical simulation and an application of rolling bearing fault diagnosis are provided to illustrate the effectiveness of the proposed adaptive SCSR technique in signal processing.

2. The step-changed SR of a bistable system

2.1. The general model

The three basic ingredients of producing SR phenomenon are a bistable or multistable system, a weak coherent input (such as a periodic signal) and a source of noise that is inherent in the system, or that adds to the coherent input. It was first introduced by Benzi et al. [1] and has been experimentally observed in various bistable systems of practical importance. The non-linear system which has been extensively exploited in the study of SR is defined by the Langevin equation as

$$\mathrm{d}x/\mathrm{d}t = ax(t) - bx^3(t) + A_0 \sin(2\pi f_0 t + \varphi) + n(t) \tag{1}$$

where a, b are real parameters, A_0 is the periodic signal amplitude and f_0 is the modulation frequency. Here, we assume that the noise $n(t)$ is zero mean, Gaussian and white, with an autocorrelation function given by $E[n(t)n(t+\tau)] = 2D\delta(t-\tau)$ where D is the noise intensity.

The system in Eq. (1) is the simplest bistable (double-well) system which describes an overdamped Brownian motion in a bistabal potential $U(x) = -ax^2/2 + bx^4/4$. The barrier height of the bistable potential in the absence of modulation and noise ($A_0 = 0$, $D = 0$) is $\Delta U = a^2/4b$ and the potential minima are located at $x_0 = \pm\sqrt{a/b}$. With $A_0 > 0$, each potential minimum is alternately raised and lowered relative to the barrier height. Bistability is lost for $A_0 \geqslant (4a^3/27b)^{1/2}$. Therefore, in the absence of an input ($A_0 > 0$, $D = 0$) the state of the system is confined to one of the two wells depending on the initial condition.

For convenience, let $\varphi \equiv 0$. When $t_0 \to +\infty$, the memory of the initial conditions gets lost and the mean value $\langle x(t)\rangle_{as}$ becomes a periodic function of time, i.e. $\langle x(t)\rangle_{as} = \langle x(x+T)\rangle_{as}$. For small amplitudes, the response of the system to the periodic input signal can be written as

$$\langle x(t)\rangle_{as} = \bar{x}\sin(2\pi f_0 t - \bar{\varphi}) \tag{2}$$

with amplitude $\bar{x}$ and a phase lag $\bar{\varphi}$ which have approximate expressions, respectively, as

$$\bar{x}(D) = \frac{A_0\langle x^2\rangle_0}{D}\frac{r_k}{\sqrt{r_k^2 + \pi^2 f_0^2}} \tag{3a}$$

$$\bar{\varphi}(D) = \arctan(\pi f_0/r_k) \tag{3b}$$