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Stochastic resonance and improvement by noise in optimal detection strategies

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Abstract

A stochastic resonance effect, under the form of a noise-improved performance, is shown feasible for a whole range of optimal detection strategies, including Bayesian, minimum error-probability, Neyman–Pearson, and minimax detectors. In each case, situations are demonstrated where the performance of the optimal detector can be improved (locally) by raising the level of the noise. This is obtained with a nonlinear signal-noise mixture where a non-Gaussian noise acts on the phase of a periodic signal.

Keywords: Optimal detectors; Noise; Stochastic resonance

1. Introduction

Stochastic resonance is a phenomenon in which some processing done on a signal can be improved by the action of the noise [1–3]. The feasibility of stochastic resonance has now been reported in a large variety of processes, under many different forms [4–9]. Yet, until very recently, stochastic resonance as an improvement of the performance by noise, was limited to suboptimal devices or processors. In the context of detection problems, various aspects of stochastic resonance have been investigated [10–18], yet with improvement by noise limited to suboptimal detection strategies. Very recently [19,20], stochastic resonance has been shown feasible also in optimal processing, in a Bayesian detection problem

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on a nonlinear signal-noise mixture with non-Gaussian noise. In the present paper we consider the same type of detection problem, and we extend the demonstration of feasibility of stochastic resonance, to a whole range of standard optimal detection strategies, in a coherent perspective.

2. Strategies for optimal detection

In this section, we briefly review standard strategies for optimal detection. Our aim here is to exhibit the definition of the optimal detectors we will be considering, and the way their performance is assessed. Classical proofs and developments can be found in [21,22]. Later on, our point will be to show that situations can be found where the performance of each one of these optimal detectors can be improved by operating them at higher noise levels, over some ranges of the noise.

In a standard detection problem, one among two known signals $s_0(t)$ or $s_1(t)$ may be mixed to a noise $\eta(t)$, the resulting mixture forming the observable signal $x(t)$. Observation of $x(t)$ at N distinct times t_k, for $k = 1$ to N, provides N data points $x_k = x(t_k)$. From the data $\boldsymbol{x} = (x_1, \ldots, x_N)$, it is to be decided whether $x(t)$ is formed by $\eta(t)$ mixed to $s_0(t)$ (hypothesis H_0) or to $s_1(t)$ (hypothesis H_1). Any conceivable detector is equivalent to a partition of $\mathbb{R}^N$ into two disjoint complementary subsets $\mathcal{R}_0$ and $\mathcal{R}_1$, such that when $\boldsymbol{x}$ falls in $\mathcal{R}_i$ then the detector decides H_i, for $i \in \{0, 1\}$.

2.1. Bayesian detection

If the prior probabilities are known, P_0 for hypothesis H_0, and $P_1 = 1 - P_0$ for H_1, it is possible to assess the performance of a given detector by means of a Bayesian cost. One introduces four elementary costs C_{ij} of deciding H_i when H_j holds, $i, j \in \{0, 1\}$, with necessarily $C_{10} > C_{00}$ and $C_{01} > C_{11}$ to penalize wrong decisions. The average Bayesian cost is then

$$C = P_0 C_{00} \int_{\mathcal{R}_0} p(\boldsymbol{x}|\mathrm{H}_0)\, d\boldsymbol{x} + P_1 C_{01} \int_{\mathcal{R}_0} p(\boldsymbol{x}|\mathrm{H}_1)\, d\boldsymbol{x} + P_0 C_{10} \int_{\mathcal{R}_1} p(\boldsymbol{x}|\mathrm{H}_0)\, d\boldsymbol{x} + P_1 C_{11} \int_{\mathcal{R}_1} p(\boldsymbol{x}|\mathrm{H}_1)\, d\boldsymbol{x}, \tag{1}$$

where $p(\boldsymbol{x}|\mathrm{H}_i)$ is the probability density for observing $\boldsymbol{x}$ when H_i holds, and $\int d\boldsymbol{x}$ stands for the N-dimensional integral $\int \ldots \int dx_1 \ldots dx_N$. The cost C of Eq. (1) is minimized by the optimal Bayesian detector that uses the likelihood ratio

$$L(\boldsymbol{x}) = \frac{p(\boldsymbol{x}|\mathrm{H}_1)}{p(\boldsymbol{x}|\mathrm{H}_0)} = \frac{\Pr\{\boldsymbol{x}|\mathrm{H}_1\}}{\Pr\{\boldsymbol{x}|\mathrm{H}_0\}} \tag{2}$$

to implement the test

$$L(\boldsymbol{x}) \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \frac{P_0}{P_1} \frac{C_{10} - C_{00}}{C_{01} - C_{11}}, \tag{3}$$