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Impact of transportation contract on inventory systems with demand cancellation

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ABSTRACT

Supply contracts often specify the quantity of inventory for shipments where retailers are liable to pay for ordering costs if order quantity exceeds the contracted size. We analyze a periodic review system where the firm manages its demand that are reserved with a one-period leadtime together with a multi-tier supply contract. We show that the optimal inventory policy has the primary structure of "finite generalized base stock" policy whose critical numbers depend on reservation parameters. The single, multiple periods and the infinite horizon models are studied. The presence of ordering costs needs a different approach from that in Yuan and Cheung (2003) to analyze the infinite horizon model.

1. Introduction

This work considers a periodic review inventory system with demand reservation and cancellation under a supply contact. All customer orders need to be reserved one period in advance, and all the reserved orders are allowed to be cancelled before their realization. There is a supply contract where a higher ordering cost is incurred whenever the quantity exceeds a certain number. The optimal inventory policies are analytically derived for single period, finite and infinite horizons, respectively, which are of the type "finite generalized base stock" policy, similar to that in Frederick (2009). The techniques in proving the policy optimality in the infinite horizon scenario in Yuan and Cheung (2003) or Yeo and Yuan (2011) are no longer applicable to the case due to the presence of the ordering cost under the supply contract. We mathematically prove the optimality of these inventory policies, particularly, for the infinite horizon scenario, and analyze the impacts of the supply contract on these optimal inventory policies. The results in the work are an interesting extension to those in Yuan and Cheung (2003).

Our work is motivated by two industrial situations involving retailers obtaining their supply of raw materials to distribute different lines of end products. The first scenario is an online computer retailer connected to an assembly plant, which in turns obtains a certain component from two suppliers: one of them is being capacitated; charges lower ordering cost of delivery via truck while the other is located offshore, incurring higher shipment cost. Ignoring transportation cost differential between the two suppliers, the firm practices an order-up-to policy at an aggregate level for this particular component. Will this ordering policy be optimal? From another perspective, this model is equivalent to the manager facing one supplier offering a single-tier supply contract where a higher ordering cost is charged if the item ordered exceeds a certain quantity. When the supplier offers a multi-tier supply contract, it is equivalent to the scenario of facing two or more capacitated suppliers. According to comScore, Inc., "computer and electronics" category is the greatest out-performer of more than 9% y.o.y (year-on-year) in e-commerce sales growth for Q2 2010. The second scenario involves the gas industry where transportation contracts play a huge role because its reserves are normally quite distant from consumer markets. Natural gas is a commodity with relatively inelastic supply due to recent efforts by countries with proven gas reserves to form a cartel, the Gas Exporting Countries Forum (GECF), to control output. In contrast to supply contract that traditionally fix the volume and price of gas over a specified period, multi-tier contract is often written to provide greater flexibility to reflect the economic value under changing conditions such as winter or output tightening by GEFC. According to NaturalGas.org, a relatively new phenomenon known as "natural gas marketing" has become an integral component of the gas industry. Such marketing activity involves coordinating the business of bringing natural gas from the wellhead to end-users. At AllConnect.com or Whitefence.com, consumers are able to obtain gas via internet marketers and there is a grace period for cancellation without incurring penalty.

Traditionally, a supply contract is a commitment that is established between two parties stretching over a long period of planning horizon. Bassok and Anupindi (1997) analyze a periodic

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review stochastic inventory model in which the buyer is committed to buying a total minimum quantity over the planning horizon. Henig et al. (1997) study a multi-period inventory-control model under a supply contract that specifies a fixed volume of inventory to deliver. During each period, ordering any quantity exceeding the contracted volume will result in a cost that is proportional to the excess borned by the retailer. They show that the structure of the optimal policy is a three-parameter policy, instead of a base stock policy. The finite and infinite horizon models are solved completely. There have been two works that extend the model considered by Henig et al. (1997): the first work is due to Chao and Zipkin (2008) who consider a fixed cost if the order quantity is above the contract volume. They partially characterize the optimal policy for the problem and propose a simple heuristic to compute the parameters of the optimal policy. Xu (2005) considers a periodic review inventory problem with supply contract allowing buyer to cancel his orders. His goal is to choose an ordering and canceling policy so as to minimize the expected cost during the planning horizon. Bassok and Anupindi (2008) consider an important class of supply contract known as the Rolling Horizon Flexibility (RHF) contracts in a multiple period setting. Under such a contract, the buyer is allowed to adjust and update its future commitment in every period. Thus, the contract represents a high level of long term and low level of short-term flexibility. They discuss a general model to incorporate adjustment flexibility, and present two heuristics, demonstrating their effectiveness but the structure of the optimal inventory policy is unknown. Lian and Deshmukh (2009) study Rolling Horizon Planning (RHP) supply contracts where the buyer is allowed to increase order amount of future orders on a rolling horizon manner, and has to pay extra cost for any extra quantities of unit ordered. They develop heuristics known as Frozen Ordering Planning (FOP) and second level Frozen Ordering Planning (FOPII). These heuristics are compared against the order-up-to policy using the objective which minimizes the total holding and penalty costs. He and Wang (2011) develop a production–inventory model for deteriorating items with demand disruption. They divide the problem into different scenarios according to disruption's time and magnitude. Optimal production and inventory plans are provided so that the manufacturer can satisfy the new demand and decrease the loss.

Our work can also be viewed as a variant of an inventory problem with multiple suppliers and limited capacity. In the literature on the inventory systems with multiple suppliers, most works focus on dual delivery modes with higher cost and shorter delivery leadtime for the emergency supplier. The pioneering work of Barankin (1961) investigates the optimal policy for dual supply sources for the single period problem. Fukuda (1964) extends his work to the multiple period case. He proves the existence of two parameters $y^0 < y^1$ such that if the stock on hand is less than y^0, then order-up-to the base stock level at the emergency mode and $y^1 - y^0$ at the regular mode, otherwise the optimal policy is a base stock policy at the regular delivery mode. The difference between the leadtimes of the expedited and regular source is one. Whittmore and Saunders (1977) study the multiple period inventory model by allowing the expedited and regular lead times to be of arbitrary length. But the form of the optimal policy is extremely complicated. Chiang and Gutierrez (1996) analyze an inventory model whose review period is larger than the supply leadtimes of both suppliers. Two types of orders can be placed at the regular review and emergency epochs. They determine the optimal policy for placing orders at the different epochs. Yang et al. (2005) consider an inventory model with Markovian in-house production capacity, facing stochastic demand and having the option to outsource. They show that the optimal outsourcing policy is always of (s,S) type and the optimal production policy is of the modified base-stock type under fairly general assumptions. Frederick (2009) develops an inventory model with multiple sources of supply. He assumes that when the initial inventory exceeds a certain critical level, the manager will return or "order-down-to" an optimal quantity of inventory at no additional cost. He proves the optimality of the "finite generalized base stock" policy for the discounted cost criterion. The mathematical model considered in his work is a generalization of Henig et al. (1997) who study a supply contract embedded in an inventory model.

The positioning of our work with respect to the existing literature is as follows. When the manager orders from a capacitated supplier while simultaneously having a more expensive unlimited supply source, this problem is mathematically equivalent to the retailer engaging in two-tier supply contract. Such equivalence entails us to extend the work of Henig et al. (1997) in an inventory model by embedding it with a two-tier level supply contract. Specifically, if the quantity ordered is greater than ν_1 but less than equal to ν_2, a unit cost of c_1 will be incurred for delivery. Furthermore, if the quantity exceeds ν_2, a unit cost of $c_2(>c_1)$ is incurred for delivery. However, we also take into account the impact of demand cancellation on the optimal replenishment policy. As a result, our analysis of the discounted cost function is bivariate in two information given: initial inventory level and number of items reserved in the previous period. When both ν_1 and ν_2 go to infinity, our model collapses to that of Yuan and Cheung (2003). It turns out that our model can easily extend the work of Yuan and Cheung (2003) with non-negative ordering costs. Technically, it is the special case of a one-tier supply contract problem when $\nu_1 = \nu_2 = 0$. Our supply contract also differs from Lian and Deshmukh (2009) who assume that unit costs for ordering each unit decreases on the rolling horizon. Furthermore, they did not focus in addressing policy optimality.

Our research yields the following insights. Firstly, much as "order-up-to" policy is popular among industries due to its simple structure, it is fact suboptimal in the presence of a supply contract and is dominated by "finite generalized base stock" policy. Moreover, the critical numbers are dependent on the reservation parameter. Secondly, we show that the structure of "order-up-to" policy is still preserved when we assume ordering costs in the work of Yuan and Cheung (2003). Without ordering cost, moral hazard on the ordering behavior is induced, and the optimal quantity in Yuan and Cheung (2003) is always greater than that by using "generalized base stock policy".

The rest of the work is organized as follows. The model and notations are developed in Section 2. Section 3 presents a model for the single period. The convexity for the optimal cost is established and the optimal ordering level is derived. Section 4 analyzes the finite horizon model. We also compare the differences of the optimal policies among our model, the model of Yuan and Cheung (2003) and the model of Yeo and Yuan (2011). In Section 5, we solve the optimal policy for the infinite horizon model. Finally, we provide a concluding note including some possible extensions to this work in Section 7.

2. Model

We consider a periodic review inventory system. Following the work of Yuan and Cheung (2003) and Yeo and Yuan (2011), all demands are made through reservations. Demands reserved in the previous periods are supposed to be fulfilled in the current period. However, customers' are allowed to cancel their reservation. Denote $\mathbf{N}$ to be the set of non-negative integers. Let D_n be the demand that is reserved during period $n \in \mathbf{N}$, and let R_n be the ratio of the demand reserved during the previous period that is