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Stochastic resonance in noisy threshold neurons

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Abstract

Stochastic resonance occurs when noise improves how a nonlinear system performs. This paper presents two general stochastic-resonance theorems for threshold neurons that process noisy Bernoulli input sequences. The performance measure is Shannon mutual information. The theorems show that small amounts of independent additive noise can increase the mutual information of threshold neurons if the neurons detect subthreshold signals. The first theorem shows that this stochastic-resonance effect holds for all finite-variance noise probability density functions that obey a simple mean constraint that the user can control. A corollary shows that this stochastic-resonance effect occurs for the important family of (right-sided) gamma noise. The second theorem shows that this effect holds for all infinite-variance noise types in the broad family of stable distributions. Stable bell curves can model extremely impulsive noise environments. So the second theorem shows that this stochastic-resonance effect is robust against violent fluctuations in the additive noise process.

Keywords: Stochastic resonance; Noise processing; Threshold neurons; Infinite-variance noise; Mutual information

1. The benefits of noise

Noise can sometimes help neural or other nonlinear systems. Fig. 1 shows that small amounts of Gaussian pixel noise improves the standard 'baboon' image while too much noise degrades the image.

Small amounts of additive noise can also improve the performance of threshold neurons or of neurons with steep signal functions when the neurons process noisy Bernoulli sequences. Several researchers have found some form of this "stochastic resonance" (SR) effect (Bulsara & Zador, 1996; Collins, Chow, Capela, & Imhoff, 1996; Collins, Chow, & Imhoff, 1995; Douglass, Wilkens, Pantazelou, & Moss, 1993; Gammaitoni, 1995; Godivier & Chapeau-Blondeau, 1998; Hess & Albano, 1998; Jung, 1995; Jung & Mayer-Kress, 1995; Stocks, 2001) when either mutual information or input–output correlation (or signal-to-noise ratio) measures a neuron's response to a pulse stream of noisy subthreshold signals. But these studies have all used simple *finite*-variance noise types such as Gaussian or uniform noise. They further assume that the noise is both symmetric and two-sided (hence zero mean). We show that SR still occurs if the noise violates these assumptions.

The two theorems below establish that the mutual-information form of the SR effect occurs for almost all noisy threshold neurons. The first theorem holds for any finite-variance noise type that obeys a simple mean condition. A corollary shows that the SR effect still occurs for right-sided noise from the popular family of gamma probability density functions. Fig. 3 shows some simulation instances of this corollary. The second theorem holds for any infinite-variance noise type from the broad family of stable distributions. All signals are subthreshold.

Infinite variance does not imply infinite dispersion. Stable probability densities have finite dispersions but have infinite variances and infinite higher-order moments. The dispersion controls the width of the bell curve for symmetric stable densities (see Fig. 4). Fig. 2 shows a simulation instance of the second theorem. Infinite-variance Cauchy noise corrupts the subthreshold signal stream but still produces the characteristic nonmonotonic signature of SR. The theorem on infinite-variance noise implies that the SR effect is robust against impulsive noise: a threshold neuron can extract some information-theoretic gain even from noise streams that contain occasional violent spikes of noise. The noise stream itself is a local form of free energy that neurons can exploit.

The combined results support Linsker's hypothesis (Linsker, 1988, 1997) that neurons have evolved to

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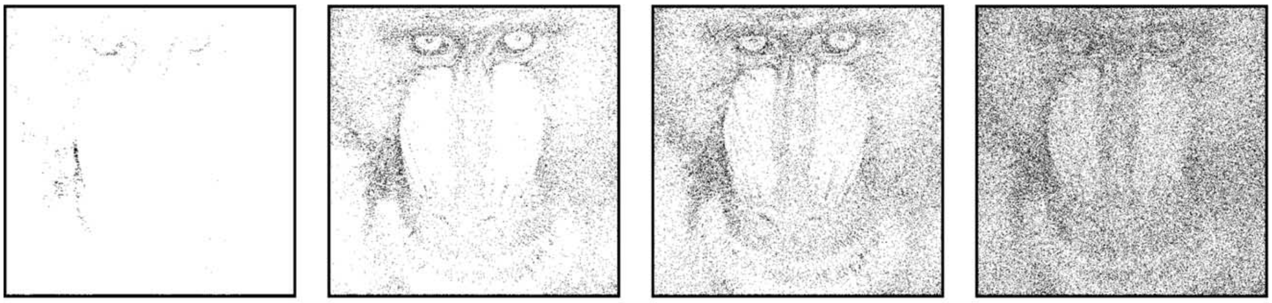


(a) (b) (c) (d)

Fig. 1. Gaussian pixel noise can improve the quality of an image through a stochastic-resonance or dithering process (Gaimmaitoni, 1995; Wannamaker, Lipshitz & Vanderkooy, 2000). The noise produces a nonmonotonic response: A small level of noise sharpens the image features while too much noise degrades them. These noisy images result when we apply a pixel threshold to the 'baboon' image. The system first quantizes the original gray-scale baboon image into a binary image of black and white pixels. It gives a white pixel as output if the input gray-scale pixel equals or exceeds a threshold θ. It gives a black pixel as output if the input gray-scale pixel falls below the threshold $\theta : y = g((x+n) - \theta)$ where $g(x) = 1$ if $x \geq 0$ and $g(x) = 0$ if $x < 0$ for an input pixel value$x \in [0,1]$ and output pixel value $y \in \{0,1\}$. The input image's gray-scale pixels vary from 0 (black) to 1 (white). The threshold is $\theta = 0.04$. Thresholding the original baboon image gives the faint image in (a). The Gaussian noise n has zero mean for images (b)–(d). The noise variance σ_n^2 grows from (b) to (d): $\sigma_n^2 = 1.00 \times 10^{-2}$ in (b), $\sigma_n^2 = 2.25 \times 10^{-2}$ in (c), and $\sigma_n^2 = 9.00 \times 10^{-2}$ in (d).

maximize the information content of their local environment. The new twist to the hypothesis is that maximizing a threshold neuron's mutual information requires deliberate use of environmental noise.

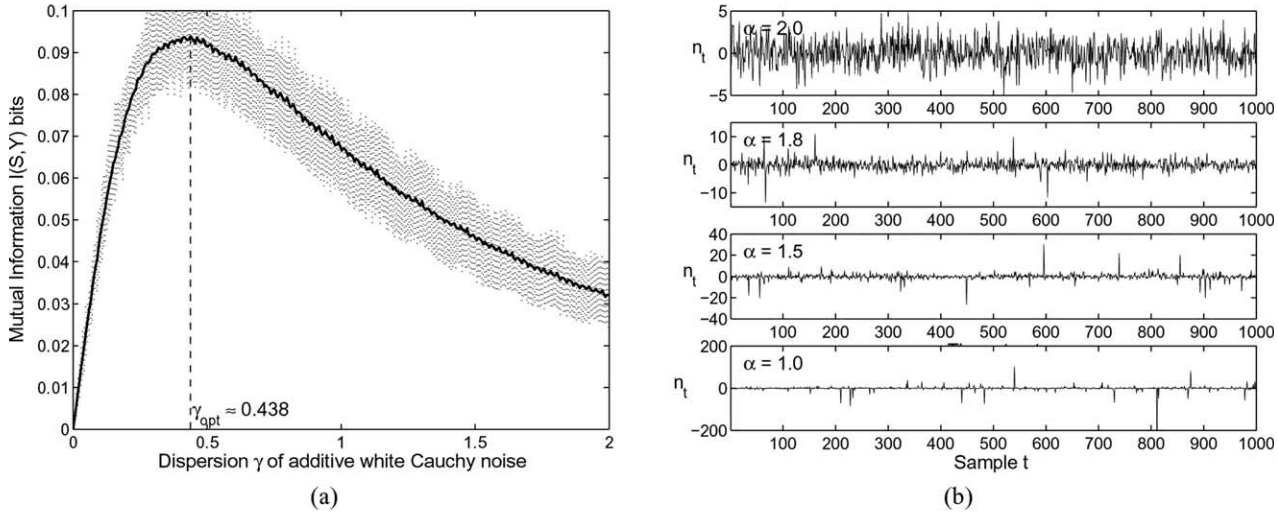
2. Threshold neurons and Shannon's mutual information

We use the standard discrete-time threshold neuron model (Bulsara & Zador, 1996; Gammaitoni, 1995; Hopfield, 1982; Jung, 1995; Kosko, 1991; Kosko & Mitaim, 2001)

$$y_t = \text{sgn}(s_t + n_t - \theta) = \begin{cases} 1 & \text{if } s_t + n_t \geq \theta \\ 0 & \text{if } s_t + n_t < \theta \end{cases} \quad (1)$$

where $\theta > 0$ is the neuron's threshold, s_t is the bipolar input Bernoulli signal (with arbitrary success probability p such that $0 < p < 1$) with amplitude $A > 0$, and n_t is the additive white noise with probability density $p(n)$.

The threshold neuron uses subthreshold binary signals. The symbol '0' denotes the input signal $s = -A$ and output signal



(a) (b)

Fig. 2. SR with infinite-variance Cauchy noise. (a) The graph shows the smoothed input-output mutual information of a threshold neuron as a function of the dispersion of additive white Cauchy noise n_t. The dispersion γ controls the width of the Cauchy bell curve. The vertical dashed lines show the absolute deviation between the smallest and largest outliers in each sample average of 100 outcomes. The neuron has a nonzero noise optimum at $\gamma_{\text{opt}} \approx 0.438$ and thus shows the SR effect. The noisy signal-forced threshold neuron has the form of Eq. (1). The Cauchy noise n_t adds to the bipolar input Bernoulli signal s_t. The neuron has threshold $\theta = 1$. The input Bernoulli signal has amplitude $A = 0.8$ with success probability $p = \frac{1}{2}$. Each trial produced 10,000 input–output samples $\{s_t, y_t\}$ that estimated the probability densities to obtain the mutual information. (b) Sample realizations of symmetric (bell-curve) alpha-stable random variables with zero location ($a = 0$) and unit dispersion ($\gamma = 1$). The plots show realizations when $\alpha = 2$,1.8,1.5, and 1. Note the scale differences on the y-axes. The alpha-stable variable n becomes more impulsive as the parameter α falls. The algorithm in (Chambers, Mallows, & Stuck, 1976; Tsakalides & Nikias, 1996) generated these realizations.