

# Stochastic resonance in binary composite hypothesis-testing problems in the Neyman–Pearson framework<sup>☆</sup>

Suat Bayram, Sinan Gezici<sup>\*</sup>

Department of Electrical and Electronics Engineering, Bilkent University, Bilkent, Ankara 06800, Turkey

## ARTICLE INFO

### Article history:

Available online 20 February 2012

### Keywords:

Binary hypothesis-testing  
Composite hypothesis-testing  
Stochastic resonance (SR)  
Neyman–Pearson  
Least-favorable prior

## ABSTRACT

Performance of some suboptimal detectors can be enhanced by adding independent noise to their inputs via the stochastic resonance (SR) effect. In this paper, the effects of SR are studied for binary composite hypothesis-testing problems. A Neyman–Pearson framework is considered, and the maximization of detection performance under a constraint on the maximum probability of false-alarm is studied. The detection performance is quantified in terms of the sum, the minimum, and the maximum of the detection probabilities corresponding to possible parameter values under the alternative hypothesis. Sufficient conditions under which detection performance can or cannot be improved are derived for each case. Also, statistical characterization of optimal additive noise is provided, and the resulting false-alarm probabilities and bounds on detection performance are investigated. In addition, optimization theoretic approaches to obtaining the probability distribution of optimal additive noise are discussed. Finally, a detection example is presented to investigate the theoretical results.

© 2012 Elsevier Inc. All rights reserved.

## 1. Introduction

Stochastic resonance (SR) refers to a physical phenomenon that is observed as an improvement in the output of a nonlinear system when noise level is increased or specific noise is added to the system input [1–15]. Although noise commonly degrades performance of a system, it can also improve performance of some nonlinear systems under certain circumstances. Improvements that can be obtained via noise can be in various forms, such as an increase in output signal-to-noise ratio (SNR) [1–3] or mutual information [8–13], a decrease in the Bayes risk [16–18], or an increase in probability of detection under a constraint on probability of false-alarm [14,15,19–21]. The first study on the SR phenomenon was performed in [1] to explain the periodic recurrence of ice gases. In that work, presence of noise was taken into account in order to explain a natural phenomenon. Since then, the SR concept has been considered in numerous nonlinear systems, such as optical, electronic, magnetic, and neuronal systems [7].

The SR phenomenon has been investigated for hypothesis-testing (detection) problems in recent studies such as [14–30]. By injecting additive noise to the system or by adjusting the noise parameters, performance of some suboptimal detectors can be improved under certain conditions [19,24]. The phenomenon

of improving performance of a detector via noise is also called noise-enhanced detection (NED) [31,32]. Depending on detection performance metrics, additive noise can improve performance of suboptimal detectors according to the Bayesian [16], minimax [20], and Neyman–Pearson [14,15,19,25] criteria. The effects of additive noise on performance of suboptimal detectors are investigated in [16] according to the Bayesian criterion under uniform cost assignment. It is proven that the optimal noise that minimizes the probability of decision error has a constant value, and a Gaussian mixture example is presented to illustrate the improbability of a suboptimal detector via adding constant “noise”, which is equivalent to shifting the decision region of the detector. The study in [20] investigates optimal additive noise for suboptimal variable detectors according to the Bayesian and minimax criteria based on the results in [14] and [16].

In the Neyman–Pearson framework, additive noise can be utilized to increase probability of detection under a constraint on probability of false-alarm. In [24], noise effects are investigated for sine detection and it is shown that the conventional incoherent detector can be improved under non-Gaussian noise. In [19], an example is presented to illustrate the effects of additive noise for the problem of detecting a constant signal in Gaussian mixture noise. In [14], a theoretical framework for investigating the effects of additive noise on suboptimal detectors is established according to the Neyman–Pearson criterion. Sufficient conditions are derived for improbability and nonimprovability of a suboptimal detector via additive noise, and it is proven that optimal additive noise can be generated by a randomization of at most two discrete signals, which is an important result since it greatly simplifies the

<sup>☆</sup> Part of this work was presented at the International Conference on Signal Processing and Communications Systems, 2009.

<sup>\*</sup> Corresponding author. Fax: +90 312 266 4192.

E-mail addresses: [sbayram@ee.bilkent.edu.tr](mailto:sbayram@ee.bilkent.edu.tr) (S. Bayram), [gezici@ee.bilkent.edu.tr](mailto:gezici@ee.bilkent.edu.tr) (S. Gezici).

calculation of the optimal noise probability density function (PDF). An optimization theoretic framework is provided in [15] for the same problem, which also proves the two mass point structure of the optimal additive noise PDF, and, in addition, states that an optimal additive noise may not exist in certain cases.

The results in [14] are extended to variable detectors in [20], and similar conclusions as in the fixed detector case are made. In addition, the theoretical framework in [14] is employed for sequential detection and parameter estimation problems in [33] and [34], respectively. In [33], a binary sequential detection problem is considered, and additive noise that reduces at least one of the expected sample sizes for the sequential detection system is obtained. In [34], improbability of estimation performance via additive noise is illustrated under certain conditions for various estimation criteria, and the form of the optimal noise PDF is derived in each case. The effects of additive noise are studied also for detection of weak sinusoidal signals and for locally optimally detectors. In [26] and [27], detection of a weak sinusoidal signal is considered, and improvements on detection performance are investigated. In addition, [28] focuses on the optimization of noise and detector parameters of locally optimal detectors for the detection of a small-amplitude sinusoid in non-Gaussian noise.

The theoretical studies in [14] and [15] on the effects of additive noise on signal detection in the Neyman–Pearson framework consider *simple* binary hypothesis-testing problems in the sense that there exists a single probability distribution (equivalently, one possible value of the unknown parameter) under each hypothesis. The main purpose of this paper is to study *composite* binary hypothesis-testing problems, in which there can be multiple possible distributions, hence, multiple parameter values, under each hypothesis [35]. The Neyman–Pearson framework is considered by imposing a constraint on the *maximum* probability of false-alarm, and three detection criteria are studied [36]. In the first one, the aim is to maximize the sum of the detection probabilities for all possible parameter values under the first (alternative) hypothesis  $\mathcal{H}_1$  (*max-sum* criterion), whereas the second one focuses on the maximization of the minimum detection probability among all parameter values under  $\mathcal{H}_1$  (*max-min* criterion). Although it is not commonly used in practice, the maximization of the maximum detection probability among all parameter values under  $\mathcal{H}_1$  is also studied briefly for theoretical completeness (*max-max* criterion). For all detection criteria, sufficient conditions under which performance of a suboptimal detector can or cannot be improved via additive noise are derived. Also, statistical characterization of optimal additive noise is provided in terms of its PDF structure in each case. In addition, the probability of false-alarm in the presence of optimal additive noise is investigated for the max-sum criterion, and upper and lower bounds on detection performance are obtained for the max-min criterion. Furthermore, optimization theoretic approaches to obtaining the optimal additive noise PDF are discussed for each detection criterion. Both particle swarm optimization (PSO) [37–40] and approximate solutions based on convex relaxation [41] are considered. Finally, a detection example is provided to investigate the theoretical results.

The main contributions of the paper can be summarized as follows:

- Theoretical investigation of the effects of additive noise in binary *composite* hypothesis-testing problems in the Neyman–Pearson framework.
- Extension of the improbability and nonimprovability conditions in [14] for simple hypothesis-testing problems to the composite hypothesis-testing problems.
- Statistical characterization of optimal additive noise according to various detection criteria.

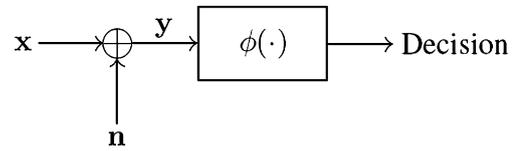


Fig. 1. Independent noise  $\mathbf{n}$  is added to data vector  $\mathbf{x}$  in order to improve the performance of the detector,  $\phi(\cdot)$ .

- Derivation of upper and lower bounds on the detection performance of suboptimal detectors according to the max-min criterion.
- Optimization theoretic approaches to the calculation of optimal additive noise.

The remainder of the paper is organized as follows. Section 2 describes the composite hypothesis-testing problem, and introduces the detection criteria. Then, Sections 3 and 4 study the effects of additive noise according to the max-sum and the max-min criteria, respectively. In Section 5, the results in the previous sections are extended to the max-max case, and the main implications are briefly summarized. A detection example is provided in Section 6, which is followed by the concluding remarks.

## 2. Problem formulation and motivation

Consider a binary composite hypothesis-testing problem described as

$$\begin{aligned} \mathcal{H}_0: & p_{\theta_0}(\mathbf{x}), \quad \theta_0 \in \Lambda_0, \\ \mathcal{H}_1: & p_{\theta_1}(\mathbf{x}), \quad \theta_1 \in \Lambda_1 \end{aligned} \quad (1)$$

where  $\mathcal{H}_i$  denotes the  $i$ th hypothesis for  $i = 0, 1$ . Under hypothesis  $\mathcal{H}_i$ , data (observation)  $\mathbf{x} \in \mathbb{R}^K$  has a PDF indexed by  $\theta_i \in \Lambda_i$ , namely,  $p_{\theta_i}(\mathbf{x})$ , where  $\Lambda_i$  is the set of possible parameter values under hypothesis  $\mathcal{H}_i$ . Parameter sets  $\Lambda_0$  and  $\Lambda_1$  are disjoint, and their union forms the parameter space,  $\Lambda = \Lambda_0 \cup \Lambda_1$  [35]. In addition, it is assumed that the probability distributions of the parameters are not known *a priori*.

The expressions in (1) present a generic formulation of a binary composite hypothesis-testing problem. Such problems are encountered in various scenarios, such as in radar systems and non-coherent communications receivers [35,42]. In the case that both  $\Lambda_0$  and  $\Lambda_1$  consist of single elements, the problem in (1) reduces to a *simple* hypothesis-testing problem [35].

A generic detector (decision rule), denoted by  $\phi(\mathbf{x})$ , is considered, which maps the data vector into a real number in  $[0, 1]$  that represents the probability of selecting  $\mathcal{H}_1$  [35]. The aim is to investigate the effects of additive independent noise to the original data,  $\mathbf{x}$ , of a given detector, as shown in Fig. 1, where  $\mathbf{y}$  represents the modified data vector expressed as

$$\mathbf{y} = \mathbf{x} + \mathbf{n}, \quad (2)$$

with  $\mathbf{n}$  denoting the additive noise term that is independent of  $\mathbf{x}$ .

The Neyman–Pearson framework is considered in this study, and performance of a detector is specified by its probabilities of detection and false-alarm [35,36,43]. Since the additive noise is independent of the data, the probabilities of detection and false-alarm can be expressed, conditioned on  $\theta_1$  and  $\theta_0$ , respectively, as

$$P_D^{\mathbf{y}}(\theta_1) = \int_{\mathbb{R}^K} \phi(\mathbf{y}) \left[ \int_{\mathbb{R}^K} p_{\theta_1}(\mathbf{y} - \mathbf{x}) p_{\mathbf{n}}(\mathbf{x}) d\mathbf{x} \right] d\mathbf{y}, \quad (3)$$

$$P_F^{\mathbf{y}}(\theta_0) = \int_{\mathbb{R}^K} \phi(\mathbf{y}) \left[ \int_{\mathbb{R}^K} p_{\theta_0}(\mathbf{y} - \mathbf{x}) p_{\mathbf{n}}(\mathbf{x}) d\mathbf{x} \right] d\mathbf{y}, \quad (4)$$

متن کامل مقاله

دریافت فوری ←

**ISI**Articles

مرجع مقالات تخصصی ایران

- ✓ امکان دانلود نسخه تمام متن مقالات انگلیسی
- ✓ امکان دانلود نسخه ترجمه شده مقالات
- ✓ پذیرش سفارش ترجمه تخصصی
- ✓ امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
- ✓ امکان دانلود رایگان ۲ صفحه اول هر مقاله
- ✓ امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
- ✓ دانلود فوری مقاله پس از پرداخت آنلاین
- ✓ پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات