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# Characterization of stochastic resonance in a bistable system with Poisson white noise using statistical complexity measures

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ABSTRACT

This paper mainly investigates the phenomenon of stochastic resonance (SR) in a bistable system subjected to Poisson white noise. Statistical complexity measures, as new tools, are first employed to quantify SR phenomenon of given system with Poisson white noise. To begin with, the effect of Poisson white noise on SR phenomenon is studied. The results demonstrate that the curves of statistical complexity measures as a function of Poisson white noise intensity exhibit non-monotonous structure, revealing the existence of SR phenomenon. Besides, it should be noted that small mean arrival rate of Poisson white noise can promote the occurrence of SR. In order to verify the effectiveness of statistical complexity measures, signal-to-noise ratio (SNR) is also calculated. A good agreement among these results obtained by statistical complexity measures and SNR is achieved, which reveals that statistical complexity measures are suitable tools for characterizing SR phenomenon in the presence of Poisson white noise. Then, the effects of amplitude and frequency of different periodic signals, including cosine, rectangular and triangular signal, on SR behavior are investigated, respectively. One can observe that, in the case of same amplitude or frequency of signal, the influence of rectangular signal on SR phenomenon is the most significant among these three signals.



## 1. Introduction

In nature, engineering and society, stochastic excitation is ubiquitous and plays an important role on the dynamical behavior of systems. In most of previous researches, it is usually modeled as Gaussian white noise process. However, for a broad class of random phenomena, such as moving loads traveling on a bridge, earthquake shake on structures, sea wave and wind action on ships, this model is not suitable. Poisson white noise, as a typical non-Gaussian stochastic excitation, which represents a sequence of independent, identically distributed pulses arriving at random times [1], is more realistic in modeling these above-mentioned impact-type loadings [2–4]. In recent years, extensive efforts have been devoted to research the dynamical properties of stochastic systems subjected to Poisson white noise. For example, Wu and Jia et al. investigated the stationary response of nonlinear systems subjected to Poisson white noise [5,6], Grigoriu et al. studied the asymptotic stability of trivial solution of linear stochastic systems driven by Poisson white noise [7], and Köylüoğlu et al. considered the reliability problem of non-linear oscillators with Poisson driven impulses [8]. Aside from that, the phenomenon of stochastic resonance (SR) on linear or nonlinear systems in the presence of Poisson white noise has also attracted researchers' attentions.

SR, in essence, is a nonlinear cooperative effect that the response of a nonlinear system with a weak periodic signal is enhanced and maximized for an optimal level of noise parameters. Since its innovative phenomenon was discovered by Benzi et al. with

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respect to the problem of periodically recurrent ice ages [9], SR has attracted considerable attention and got wide developments and applications in the fields of science and technology [10–25], such as electronic circuits [10], soft matter systems [13,14], microcavity polaritons [16], complex networks [18], neural networks [20,21] and so on. Meanwhile, several approaches have been proposed to quantify SR phenomenon. For example, McNamara et al. introduced the signal-to-noise ratio (SNR) to characterize SR in adiabatic limit [10,11]. Zhou et al. adopted the probability density of residence times as a tool for measuring SR effect [22]. Recently, statistical complexity measures, as new and feasible indicators, were developed by Rosso et al. for detecting SR behavior of stochastic systems [23–25]. However, to the author's knowledge, few researchers focused on SR phenomenon in stochastic bistable systems excited by Poisson white noise.

It is well known that the existence of periodic force can also affect dynamical behavior of stochastic systems. In the past, periodic force in many studies are chosen as sine or cosine signal. However, for the realistic models in physics and engineering, other types of periodic forces also need to be investigated. Thus, it is necessary to study the evolution of stochastic systems with different periodic signals. So far, there are lots of efforts on the SR phenomenon in stochastic systems in the presence of different periodic forces [26–30]. Particularly, SR phenomenon in two-state model of membrane channel with dichotomous noise was studied by Ginzburg et al. in the condition of square-wave periodic signal, and the explicit expressions for the spectral density, the SNR and the coherence function were obtained in a wide range of parameters [26]. A theoretical and experimental research of SR problem in an asymmetric bistable system excited by additive white noise was reported by Gerashchenko in the presence of a periodic rectangular signal [27]. The SR phenomenon in two coupled overdamped anharmonic oscillators with Gaussian noise driven by six different external periodic forces was investigated, and the effect of each force on SR was discussed separately [29].

Motivated by the above-mentioned discussions, one can see that there are few results reported on SR problem of stochastic bistable systems under Poisson white noise and different periodic signals. In the present paper, we intend to research the influences of Poisson white noise and different periodic signals on SR phenomenon in a symmetric bistable system by using the statistical complexity measures, respectively. The organization of this paper is as follows: first, a description of Poisson white noise is given in Section 2. Second, a stochastic symmetric bistable system driven by Poisson white noise is described, and the influence of Poisson white noise on first passage time statistics of given system is investigated numerically in Section 3. Third, in Section 4, the effects of noise and signal parameters on SR phenomenon of the bistable system are detected by means of statistical complexity measures and SNR in detail, when different periodic signals are added. Finally, the conclusions are drawn in Section 5.

## 2. Poisson white noise

In this section, Poisson white noise is introduced in detail. This noise $\xi(t)$ can be treated as the formal derivative of compound Poisson processes [31]

$$\xi(t) = \frac{dC(t)}{dt} = \sum_{i=1}^{N(t)} Y_i \delta(t - t_i), \quad C(t) = \sum_{i=1}^{N(t)} Y_i U(t - t_i), \tag{1}$$

where $N(t)$ represents a Poisson counting process with mean arrival rate $\lambda > 0$, $\delta(\cdot)$ is the Dirac delta function, $U(\cdot)$ is the unit step function. The symbols $Y_i$ are independent identically distributed random variables denoting the impulse amplitude, which are independent of pulse arrival time $t_i$. The $k$th correlation function of Poisson white noise $\xi(t)$ satisfies the form [32]

$$R^{(k)}[\xi(t_1)\xi(t_2), \cdots, \xi(t_k)] = \lambda E[Y^k]\delta(t_2 - t_1)\cdots\delta(t_k - t_1), \quad k = 2, \ldots, \infty, \tag{2}$$

in which $E[\cdot]$ represents mathematical expectation. In the following section, the amplitude $Y_i$ of random impulse is assumed to be Gaussian distributed with zero mean for illustrative purpose. Obviously, the mean of Poisson white noise is zero. In the condition of $k = 2$, $D = \lambda E[Y^2]$ is known as the intensity of Poisson white noise. For the limiting case, when $\lambda$ approaches infinity and the intensity $\lambda E[Y^2]$ keeps a constant value, the Poisson white noise $\xi(t)$ tends to a Gaussian white noise $\xi'(t)$ satisfying statistical properties $\langle \xi'(t) \rangle = 0, \langle \xi'(t)\xi'(s) \rangle = D\delta(t - s)$ [33].

In Fig. 1, the time series of Poisson white noise $\xi(t)$ defined in Eq. (1) and Gaussian white noise $\xi'(t)$ are illustrated as fixing the noise intensity $D = 0.5$. The numerical algorithm of the generation of Poisson white noise is employed in Ref. [33]. One can observe clearly that, for fixed noise intensity, the amplitude of Poisson white noise decreases and the number of noise pulses increases as increasing the value of mean arrival rate. Namely, the non-Gaussian structure of Poisson white noise is reflected fully with small mean arrival rate.

## 3. Model system and first passage time statistics

Let us consider a one-dimensional symmetric bistable system driven by Poisson white noise, which is described by the following equation:

$$\frac{dx(t)}{dt} = x(t) - x^3(t) + \xi(t), \tag{3}$$

where $\xi(t)$ is Poisson white noise defined in Eq. (1) with zero mean and intensity $D = \lambda E[Y^2]$. According to Eq. (3), it is easy to obtain the system potential function $U(x) = -x^2/2 + x^4/4$, which has two stable states $x_{s1} = -1$, $x_{s2} = 1$ and the unstable point $x_{un} = 0$.