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Simulation of circuits demonstrating stochastic resonance

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Abstract

In certain dynamical systems, the addition of noise can assist the detection of a signal and not degrade it as normally expected. This is possible via a phenomenon termed stochastic resonance (SR), where the response of a nonlinear system to a subthreshold periodic input signal is optimal for some non-zero value of noise intensity. We investigate the SR phenomenon in several circuits and systems. Although SR occurs in many disciplines, the sinusoidal signal by itself is not information bearing. To greatly enhance the practicality of SR, an (aperiodic) broadband signal is preferable. Hence, we employ aperiodic stochastic resonance (ASR) where noise can enhance the response of a nonlinear system to a weak aperiodic signal. We can characterize ASR by the use of cross-correlation-based measures. Using this measure, the ASR in a simple threshold system and in a FitzHugh–Nagumo neuronal model are compared using numerical simulations. Using both weak periodic and aperiodic signals, we show that the response of a nonlinear system is enhanced, regardless of the signal.

Keywords: Circuits simulation; Stochastic resonance; Nonlinear system

1. Introduction

Noise is usually considered a nuisance in communication and signal processing systems, but via a phenomenon known as stochastic resonance (SR) noise, can assist the detection of a signal. Using the signal-to-noise ratio (SNR) as a measure of the coherence of the output signal, the signature of SR is a sharp increase in the SNR followed by a gradual decrease as the noise is increased. The three main ingredients usually required to observe SR are noise (with correlation time much lower than that of the system), subthreshold periodic signal and a nonlinear system. The nonlinear system is essential since the output would be defined by linear response theory for a linear system, thus the SNR would be proportional at the input and output of such a system. The simplest way to provide a nonlinear system is by introducing a threshold element.

Since its emergence as an explanation for the periodic recurrences on the Earth's climate [1–4] where the term SR was first coined, SR has traversed many disciplines. These range from electronic systems [5,6], sensing neurons [7,8], visual perception [9–12], bidirectional ring lasers [13] and super conducting quantum loops (SQUIDS) [14] to name a few. For further background, Gammaitoni et al. have written an extensive review [15]. More recently, SR is believed to assist with hearing systems in the auditory nerve [16–18] while adaptive systems can learn to add the optimal amount of noise to some nonlinear feedback systems [19].

As an example of SR, we will consider the work by Simonotto et al. [9], which deals with the human visual system. This is closely related to the dithering effect [10]. Consider a system that is capable of transmitting single bits of information, each of which marks a threshold crossing. A visual realization of this is shown in Fig. 1 that was generated following the procedure in Ref. [9]. The original gray scale image shown in Fig. 1a is depressed beneath a threshold, white noise added to the gray value in each pixel, and the result compared to the threshold. Thus, the noise is incoherent with that in all other pixels. Pixels with a value above the threshold are made black and the others below are made white. Every pixel contains one bit of information, whether or not the threshold has been crossed. Fig. 1b–d shows the result of adding noise of three intensities, increasing from left to right. There is an optimal noise intensity in (b) that maximizes the information content. Additional improvement in perceived picture quality can be gained by varying the noise temporally [9]. The images in Fig. 1 have been averaged over five ensembles.

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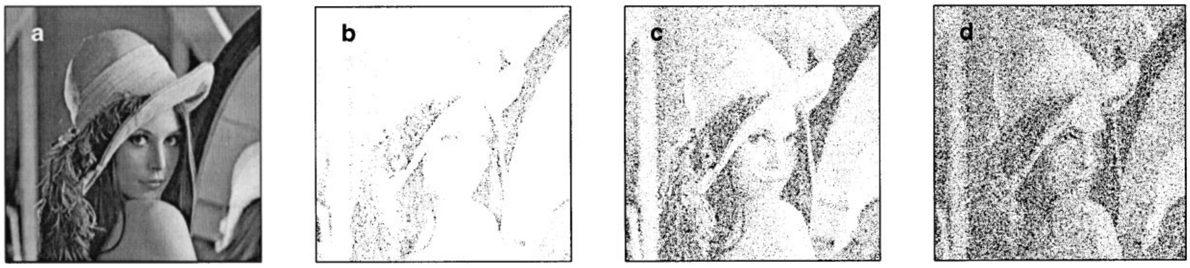

Fig. 1. The pictures are generated from the popular signal processing image of "Lenna" on a 256 gray scale color depth at a resolution of 256 × 256 pixels : (a) original image; (b)–(d) increasing noise intensities from left to right.

A limitation of SR is that it only considers periodic signals; this shortcoming has lead to the development of a method for characterizing SR with aperiodic stimuli [20], where the term aperiodic stochastic resonance (ASR) was coined. Most of the literature regarding ASR to date has considered neuronal models [20–27].

In this paper, we first describe the types of nonlinear systems and noise that are used. This is followed by algorithms used for numerical simulations. The next two sections replicate SR and ASR, which includes applying ASR to the simplest nonlinear system.

2. Nonlinear systems and noise

We used noise given by the Ornstein–Uhlenbeck (OU) stochastic process of the form

$$\dot{\zeta}(t) = \lambda\zeta(t) + \lambda\xi(t), \tag{1}$$

where $\xi(t)$ is the white Gaussian noise with mean $\langle\xi(t)\rangle = 0$ and autocorrelation $\langle\xi(t)\xi(s)\rangle = 2D\delta(t-s)$. The angled brackets $\langle\cdot\rangle$ denote an ensemble average. The correlation time of the OU process is $\tau_c = \lambda^{-1}$ and the autocorrelation is given by

$$\langle\zeta(t)\zeta(s)\rangle = \frac{D}{\tau_c}\exp\left(\frac{-|t-s|}{\tau_c}\right), \tag{2}$$

with a variance of D/τ_c. The OU process provides control over both noise intensity D, and correlation time τ_c.

In biological systems, SR has shown to be observed in sensory neurons, hence we use a neuron model as a basis for our nonlinear system. The dynamics of the FitzHugh–Nagumo (FHN) neuronal model provide a simple representation of the firing dynamics of sensory neurons [7,29]. We consider the FHN model given by the following system that is subjected to a subthreshold signal $S(t)$, and noise given by Eq. (1) [7,30]

$$\epsilon\dot{v} = v(v-a)(1-v) - w + A + S(t) + \zeta(t), \tag{3}$$

$$\dot{w} = v - w - b, \tag{4}$$

where $v(t)$ is a fast (membrane potential voltage) variable and $w(t)$ is a slow (recovery) variable. The parameters are chosen from Refs. [20,31], namely, $A = 0.04$, $\epsilon = 0.005$, $a = 0.5$, $b = 0.15$.

The characteristics of the neuron model are shown in Fig. 2, the left half is noiseless with a suprathreshold signal while the second half is purely noisy. When the sum of the time varying inputs exceeds a threshold that is determined by parameters in the FHN model, the fast membrane potential quickly increases to an excited state. Once the neuron "fires", it resets itself after a short refractory period. When this firing crosses an arbitrary threshold (set to 0.5 from Refs. [7,31], a δ-function spike is produced. These are shown in Fig. 2 by the vertical lines above the firing periods. Hence, the output of the FHN neuronal model is a train of action potentials.

Although the FHN is only a simple neuron model, it involves stochastic differential equations (SDEs), which require care when numerically simulating. A simpler alternative is to use a threshold system as a crude approximation of a neuron model. This not only simplifies the simulations but also the implementation when realizing a system in hardware. Some of the different threshold systems are as follows.

A simple level crossing circuit (LCC) was simulated, based on a single operational amplifier (op amp) with an appropriate threshold voltage applied at the inverting terminal. Whenever the signal exceeds the threshold, a high is given at the output, correspondingly, a subthreshold signal generates a low output. Essentially, we have a comparator, where the output consists of a train of variable width pulses.

Adding some resistors in the positive feedback path gives rise to a comparator with hysteresis, that is, a Schmitt trigger

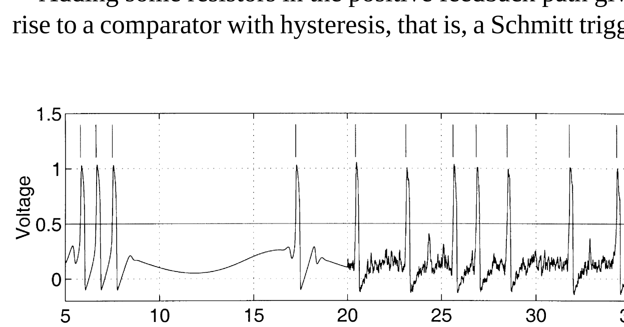

Fig. 2. The first half of the FHN neuron response (fast voltage, $v(t)$) is due to a suprathreshold signal, while the right half of the response is driven by pure noise. The output is the spike train shown by the vertical lines.