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An analytical method to solve the probabilistic load flow considering load demand correlation using the DC load flow

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ABSTRACT

Probabilistic load flow (PLF) analyses the probability behaviour of the power injected and demanded in an electrical network, providing cumulative results of the power flowing through its lines. However, the probability behaviour analysis is not complete if the correlation between load demands has not been considered. Environmental or social factors can lead to such correlation.

This paper proposes an analytical method to solve the PLF considering load demand correlation. It is based on the use of cumulants and Gram-Charlier expansion. The development proposed in the paper has been proved, the consistency of the method has been checked and the influence of the correlation in the solution of the PLF is shown.

1. Introduction

Electrical networks have been analyzed from a probabilistic point of view since 1974, when Borkowska proposed the Probabilistic Load Flow (PLF) [1]. Over the years that followed Allan et al. proposed methods to solve the PLF [2–12]. In those days, the methods were based on the convolution of Probability Density Functions (PDF) of several variables, in order to obtain the PDF of their sum. After that, the problem was solved by means of linearization [5,6,8,10,13,14] or other approximations [15,16]. Lately, there has been widespread use of cumulants and the Gram-Charlier expansion [17–19] and also the Point Estimate Method (PEM) [14,20,21]. In addition to analytical methods, Monte Carlo (MC) simulation has also been used [13,22,23]. The PLF has been applied to management, and also to short-term and long-term electrical network planning.

The methods proposed to solve the PLF consider the probabilistic nature of the injected power and the load demand. However, in most cases the correlation between the different loads has not been taken into account. The few exceptions are based either on numerical methods applied to correlated data [24–26] or on analytical methods such as the PEM [27]. The numerical methods need increased computing time because the correlated data must be obtained initially. The PEM based method becomes less accurate as the correlation increases. The correlation between the aggregated load demands connected to different buses of an electrical network is due, in part, to meteorological or social factors and constitutes an issue that can be considered when analyzing the PLF. For example, during a heat wave, most air conditioning systems would be in operation, raising total power consumption. On the other hand, most of the load demands during the night would descend to their lowest values for the previous 24-h period. Therefore, neglecting the correlation from the PLF can provide wrong results for the problem in most cases. It has been included in the methods proposed in [10,11,13].

The method proposed in [11] consists of obtaining the PDF of each input variable subject to the correlation with others whose values are fixed, and solving the PLF by applying convolution between these PDFs. In most cases, the results obtained with this analytical method do not fulfil the requested correlation values. The reason is that the input data in this case are not correct, each PDF obtained is a particular case, established by the values given to the rest of variables.

In [10], the linear relationship between correlated variables was used, leading to good results when there is high correlation, although there is not a good approximation when the correlation is around 0.5 or lower. A slight modification of this method was introduced in [13] but the same drawbacks are present there.

From this analysis of previous methods, it can be said that there is no analytical method that takes into account the correlation between load demands to gain accurate results within reasonable computer time.

This paper proposes a new method to solve the PLF by considering the correlation between aggregated load demands of different

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buses in an electrical network. It is based on the use of cumulants and their properties, and Gram-Charlier expansion.

The paper is structured as follows: Section 2 briefly introduces the theoretical developments used to solve the PLF; Section 3 develops the proposed method, Section 4 shows the application of the method proposed to the IEEE One Area RTS-96 power system; and Section 5 states the conclusions.

2. Theoretical background

2.1. Probabilistic load flow

The probabilistic nature of injected power and load demand has to be considered when analyzing LF [28–30]. The PLF was proposed to consider these types of uncertainties. Therefore, the relationships between voltage and power have to be solved considering their associated probabilistic distributions, instead of deterministic values.

Injected powers and load demands are represented by their PDF or Cumulative Distribution Function (CDF) to show their behaviour. In this paper the former has been used.

2.1.1. Injected power

Generators can be probabilistically described by means of a Bernoulli distribution where the probability of the generator being off is defined as its Forced Outage Rate (FOR). Therefore, the PDF of each generator has the Probability Mass Function (PMF) expressed below:

$$f(P_g) = \begin{cases} p & P_g = P_r \\ 1-p & P_g = 0 \end{cases} \quad (1)$$

where P_g is the power generated, P_r is its rated power, and $1-p$ is the FOR of the generator.

Notice that several generators are grouped in a power plant, which is connected to a single bus.

When other types of generator, such as solar panels or wind turbines, are considered, the output power has to be described by a continuous distribution that depends on the primary source.

2.1.2. Load demand

The aggregated load connected to a bus can be probabilistically described by a Normal distribution. Its corresponding PDF is the following:

$$f(P_l) = \frac{1}{\sqrt{(2\pi)}\sigma} \exp\left(-\frac{(P_l-\mu)^2}{2\sigma^2}\right) \quad (2)$$

where P_l is the load demand, μ is its mean value and σ is the standard deviation.

The solution of the PLF can be obtained through numerical or analytical methods. Numerical methods are based on simulation techniques such as MC, where the computation process makes great demands in terms of time. There are various types of analytical methods although the most common at present is based on the use of cumulants and the Gram-Charlier expansion [17–19].

3. Cumulants and moments

The cumulants [17,31] of a probability distribution are defined through the cumulant-generating function and provide an alternative to the moments. Their use, in some cases, helps to simplify theoretical problems.

The relationship between cumulants and raw moments, or moments about the origin, is expressed here:

$$\begin{aligned} \kappa_1 &= \mu'_1 \\ \kappa_i &= \mu'_i - \sum_{j=1}^{i} \binom{i-1}{j-1} \kappa_j \mu'_{i-j} \quad 2 \le i \end{aligned} \quad (3)$$

where μ'_i is the raw moment of order i and κ_i is the cumulant of order i.

Notice that this relationship can easily be inverted as shown below:

$$\mu'_i = \kappa_i + \sum_{j=1}^{i-1} \binom{i-1}{j-1} \kappa_j \mu'_{i-j}. \quad (4)$$

Another useful relationship can be established between central moments and raw moments, expressed here:

$$\mu_i = \sum_{j=0}^{i} (-1)^j \binom{i}{j} (\mu'_1)^j \mu'_{i-j} \quad (5)$$

where μ_i is the central moment of order i.

As has been explained above, the injected power of a generator is defined by a Bernoulli distribution, so it will be helpful to know the cumulants of this distribution. The raw moments of a Bernoulli distribution are obtained through:

$$\mu'_i = pP_r^i \quad (6)$$

Using the above equations, the cumulants of a Bernoulli distribution can easily be obtained.

The cumulants of the probability distribution of a Power Plant can be obtained by using the properties of the cumulants:

$$\kappa_i^{(b_i)} = \sum_{j=1}^{n_g} \kappa_i^{(g_j)} \quad (7)$$

where g_j represents the jth generator and b_i the ith bus.

It also will be helpful to know the cumulants of a Normal distribution, which represents the aggregated load demand in a bus. They are expressed thus:

$$\begin{aligned} \kappa_1 &= \mu'_1 = \mu \\ \kappa_2 &= \mu'_2 - \mu'^2_1 = \sigma^2 \\ \kappa_i &= 0 \quad 3 \le i \end{aligned} \quad (8)$$

where μ and σ are the parameters of the Normal distribution.

3.1. Gram-Charlier type A series

The Gram-Charlier expansion [17,31] establishes the relationship between the probability distribution of any variable with the distribution of a standard Normal one and its derivatives, for which it uses some coefficients in a similar way to the way Taylor series use them, such as can be seen below:

$$f(x) = \sum_{i=0}^{\infty} c_i H_i(x) \alpha(x) \quad (9)$$