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Point estimate method for probabilistic load flow of an unbalanced power distribution system with correlated wind and solar sources

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ABSTRACT

Context: The electric parameters of the power networks are usually analysed through deterministic power flows; however, the variation in load demands and power fluctuation of renewable generators cannot be considered with the deterministic power flows because it uses specific power values. The probabilistic power flow methods are better for this purpose since they apply techniques to include and reflect the uncertainty of input variables on the results obtained.

Objective: This paper extends the Point Estimate Method (PEM) applied to the probabilistic power flow of an unbalanced power distribution system with dispersed generation and variable power factors. This method is applied to include uncertainties of loads and power sources such as wind and solar. As PEM requires independent input random variables, but usually there is spatial correlation between loads or power sources; therefore, Cholesky decomposition is applied to deal with this situation.

Method: In this paper are combined the scheme 2m+1 of the Point Estimate Method with the Cholesky decomposition and some approximation methodologies to estimate the cumulative distribution function of some electrical parameters.

Results: The results obtained are the moments about the mean of the output variables, which are used in conjunction with some approximation methodologies to obtain an estimation of the Cumulative Distribution Function for nodes or branch parameters. The proposed methodology is tested on the three-phase unbalanced IEEE 123-node test system, and results are compared with those obtained from the benchmark Monte Carlo simulation.

Conclusions: There are comments on some pertinent information about Point Estimate Method performance on this kind of power systems.

Introduction

One section of the electric power system that can be most involved in the future of renewable installations is that which corresponds to its power distribution. The introduction of renewable power sources creates new challenges in these systems, including those posed by the spatially correlation of the sources and the variability on the power supply, which is caused by the randomness of the sources. The voltage and power profiles in the system change as a function of the power sources, along with variability in loads. Currently, the power does not travel only from the substation to the loads, nor do generators with intermittent sources always deliver the same power.

The electric parameters of the power networks are usually analyzed through deterministic power flow method (DPF); however, the variation in load demands and power fluctuation of renewable generators cannot be considered with the DPF method because it uses specific power values. The probabilistic power flow (PPF) methods are better for this purpose since they apply techniques to include and reflect the uncertainty of input variables on the results obtained.

The Monte Carlo simulation (MCS) methodology is usually the reference for probabilistic power flows because it uses nonlinear power equations and is simple to develop. This methodology, however, is demanding in terms of time and computational resources as a result of the many deterministic power flows needed to reach

Abbreviations: CC, coefficient of correlation; CV, coefficient of variation; CDF, cumulative distribution function; DPF, deterministic power flow; MCS, Monte Carlo simulation; PDF, probability density function; PEM, Point Estimate Method; PEM-CG2, second order Gram–Charlier expansion; PEM-CF1, first order Cornish–Fisher approximation; PEM-CF2, second order Cornish–Fisher approximation; PEM-ED1, first order Edgeworth expansion; PEM-ED2, second order Edgeworth expansion; PEM-N, normal approximation; PPF, probabilistic power flow.

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Nomenclature

Letters

$C_{q,h}$	covariance matrix of transformed input random variables (q) that must be equal to identity matrix
$C_{x,h}$	covariance matrix of independent input variables (x)
$E(Z^j_{l,h})$	statistical moment of order j, for output variable Z
$f(x)$	density probability function
$F(x)$	distribution probability function
G	multiplicative factor depending on probability function
h	it could be 1, 3, 5 for real power, and 2, 4, 6 for reactive power of phases A, B, and C respectively
i	concentration points for PEM $2m+1$ scheme, $i = 1,\ldots,3$
j	order of moment of an output random variable
k_3, k_4	cumulants of third and fourth order respectively
k^*_3, k^*_4	standardized cumulants of third and fourth order
l	element of the system (node, line, etc.)
L_h	lower triangular matrix of value h
m	number of independent input random variables
$M'_3(x_{k,h})$	third moment about mean of a input random variable
$M'_4(x_{k,h})$	fourth moment about mean of a input random variable
P	real power
$p_{0,h}$	totalized weight for all concentration point $i = 3$ of all input random variables
$p_{k,h,i}$	weight of concentration point i of input random variable k value h
P_{r_k}	nominal power of wind turbine associated to k variable
Q	reactive power
$q_{k,h,i}$	transformed input random variable k value h at concentration point i
v	it describes correlation (absence/presence) between phases, and real and reactive power
V_{ci_k}	cut in speed of wind turbine associated to k variable
V_{r_k}	nominal speed of wind turbine associated to k variable
V_{co_k}	cut-out speed of wind turbine associated to k variable
W_h	vector of independent standard normal samples, one for each input random variable
x_k	independent input random variables for $k = 1,\ldots,m$, include the set of $h = 1,\ldots,6$ values
$x_{k,h}$	independent input random variable k value h
$x_{k,h,i}$	independent input random variable k value h at concentration point i
Z	output random variable, e.g.: voltage, current, etc.
Z_h	vector of correlated standard normal samples
$Z_{l,h}(k,i)$	parameter obtained from deterministic power flows when it is used the variable k at concentration point i

Symbols

α	quantile of a distribution function
$\sigma_{k,h}$	standard deviation of variable k value h
$\varepsilon_{\delta\text{-}95}$	absolute angle error for P_{95} in degrees
ε_{Pi} (%)	relative real power error at lines in %
$\varepsilon_{Pi\text{-}95}$ (%)	relative real power error at lines for P_{95} in %
ε_{Qi} (%)	relative reactive power error at lines in %
$\varepsilon_{Qi\text{-}95}$ (%)	relative reactive power error at lines for P_{95} in %
ε_{Pij} (%)	relative real power error at nodes in %
$\varepsilon_{Pij\text{-}95}$ (%)	relative real power error at nodes for P_{95} in %
ε_{Qij} (%)	relative reactive power error at nodes in %
$\varepsilon_{Qij\text{-}95}$ (%)	relative reactive power error at nodes for P_{95} in %
ε_V (%)	relative voltage error in %
$\varepsilon_{V\text{-}95}$ (%)	relative voltage error for 95th percentile (P_{95}) in %
γ_1	skewness coefficient of variable to be approximate
γ_2	kurtosis coefficient of variable to be approximate
$\phi(x)$	standard normal density function
$\Phi(x)$	standard normal distribution function
$\lambda_{k,h,3}$	skewness coefficient of variable k value h
$\lambda_{k,h,4}$	kurtosis coefficient of variable k value h
$\lambda q_{k,h,3}$	skewness coefficient of variable q_k value h
$\lambda q_{k,h,4}$	kurtosis coefficient of variable q_k value h
$\mu_{k,h}$	mean of variable k value h
μ_k	set of mean values for variables $k = 1,\ldots,m$. this set includes the $h = 1,\ldots,6$ values
μq_h	vector of mean values of all variable q value h
μx_h	vector of mean values of all variable x value h
$\mu'_2(Z_{l,h})$	second moment about mean of an output variable
$\mu'_3(Z_{l,h})$	third moment about mean of an output variable
$\mu'_4(Z_{l,h})$	fourth moment about mean of an output variable
$\rho_x(i,j)$	correlation coefficient between variables i and j
$x(\alpha)$	inverse probability function for quantile α
$x(\alpha)^*$	inverse standard probability function for quantile α
$\xi(\alpha)$	inverse standard normal distribution for quantile α
$\xi_{k,h,i}$	factor involving skewness and kurtosis coefficients for input variable k value h at concentration point i

a convergence. The probabilistic power flow based on MCS is known as numerical method. Analytical methods—including linearization, multi-linearizations, cumulants and quadratic probabilistic load flow—are another way of solving PPF problems; these methods work with the probability density functions (PDF) or the statistics of input random variables. They require some assumptions to deal with dependence between input random variables and the nonlinearity of the power equations. Point Estimate Method (PEM) is another option and involves working with statistics of random variables, but it does not require the complete knowledge of the density functions. PEM gives an approximation of the raw moments of output variables. Moreover than PPF, methods like fuzzy logic or interval analysis can be used to solve the power flow problem when uncertainties are presented. These methods supposed an incomplete knowledge of input random variables and assume possibilities more than probabilities for them; thus, the output variables are described by ranges of possible values. The referred methodologies and their application to the power flow problem can be found in review papers as [1,2].

Of all methodologies mentioned above, PEM is one of the most recent to be evaluated in the PPF problem, and it is found to provide convenient results and performance. PEM is used to approximate the first moments of the random output variables of interest with only a few evaluations. This method has been modified since it was first proposed, and, depending on the selected scheme, some of its advantages are, among others: Good results obtained from a small sample of deterministic power flows, the possibility of using nonlinear power equations, and the ability to process correlated random variables using some additional methodologies in conjunction with PEM.

Literature review

PEM has been modified since Rosenblueth presented its 2^m method [3]. These modifications have resulted in several methods like: $2m$ method from Harr, $O(m^3)$ method from Li, and Km or $Km+1$ schemes from Hong [4], all these are described and their differences compared in [5]. As m is equal to the number of input random variables, the number of evaluations required to achieve a solution is then 2^m, $2m$, $O(m^3)$, Km or $Km+1$ respectively (with $K = 2,\ldots,4$). As it is can be observed and especially if m is greater,