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# Probabilistic load flow with correlated wind power injections

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ABSTRACT

The non-dispatchable nature of wind generation implies that system operation depends on wind power prediction programs that forecast wind farms production with high levels of uncertainty. This means that probabilistic power analysis tools become more and more necessary in systems with high wind penetration. Probabilistic load flow becomes especially difficult when wind generation is considered. The high uncertainty of the production, the non-Gaussian probability density function (PDF) and the clear dependence among the wind farms poses a challenge for conventional tools. The paper proposes an approximation that makes use of the properties of statistical moments and Cornish–Fisher expansion to tackle these new problems.



## 1. Introduction

The great proliferation of intermittent generation in power networks has increased the uncertainty in power systems. This uncertainty affects both the long and medium term system planning, and the day-ahead operation. This is why the importance of probabilistic tools for power system analysis is growing. For long-term studies, to consider the uncertainty in power planning may lead to a less expensive and more secure network. For the daily operation, an adequate assessment of the system variables may lead to a better management of congestions, among other important advantages.

Probabilistic power flow is one of the best-known probabilistic analysis tools. To study all possible combinations of generation and load is impractical due to the great size of real networks, and this is why analytical methods should be used to adequately assess the variability of the grid variables. From the first proposals in 1970s [1,2], a great deal of literature can be found about this subject. In these early references, it was the uncertainty of the load what was considered.

The most straightforward method of solving this problem is Monte Carlo simulation. This technique involves repeated simulation with values obtained from the probability density function (PDF) of the random variable considered. But for an adequate representation, a great deal of simulations must be considered in real systems. One of the alternatives to this is the convolution of the PDF of the random variables involved, when they are independent of each other, and linearly related. Although this reduces the computational burden, it is still costly to obtain the PDF of a single line when several random power injections are considered. Fast Fourier Transform (FFT) techniques were proposed to reduce the computational burden [3], but this method is linked to the convolution technique, and does not solve the problem efficiently. The use of cumulants and the approximation of resultant PDF by orthogonal series (Gram Charlier expansion series) have also been proposed [4]. It has interesting properties, and is computationally inexpensive. Another recent proposal is the point estimate method [5] that approximates the moments of the system variables of interest. A series expansion is apparently used for obtaining the PDF in this paper. All these approaches assume that the random variables considered are uncorrelated. The main drawback of point estimate method is that it needs a very complex formulation if correlation between random variables is considered.

Probabilistic load flow has mostly included the uncertainty of load. This uncertainty is not very high, and can be modeled using Gaussian probabilistic density functions. Wind energy proliferation, however, poses new challenges, since the variability of wind power production is much higher, and usually the PDF are not Gaussian. Long-term planning studies must consider PDF based on Weibull distributions, while short-term operation analysis need to use PDF whose estimation is still under study. Reference [6] proposes the use of FFT and convolution in distribution networks, and makes a simplified estimation of the PDF for short-term wind power prediction.

Dependence between random variables must be inexcusably considered when wind farms production is to be included, since the power injected by one farm is clearly correlated to other farms of the same area. Dependence between loads' uncertainty is also clear, because their variations are normally due to similar causes. This dependence has been considered, only between loads, in [7], where it is modeled with a linear relation, and in [8], where the covariance has been taken into account in the equations.

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For large transmission networks it seems that the approach based on cumulants is very adequate due to the low computational requirements, and its possibility to be easily generalized for dependent random variables. However, for non-Gaussian PDF, Gram Charlier expansion series have serious convergence problems, and other approaches, such as Cornish–Fisher expansion, give better results, without more computational burden [13].

This paper proposes an analytical way of solving the probabilistic load flow problem, when non-Gaussian functions and dependence between random variables are to be considered. A combination of methods is used in order to reach a maximal accuracy with a minimal computational cost. A $(2m+1)$ point estimate method is used to find the grid variables (power flows and voltages), a linear approximation is used to obtain higher order moments of these variables, taking into account the dependence, and Cornish–Fisher expansion is used to approximate the cumulative distribution functions (CDF) of grid variables, and specially the quantiles of the lines of interest. All these methods produce a successful approach.

Although this method is general, it is applied, in this paper, to the next day operation of power systems with great wind penetration. Usually, the system operators of these networks use the output of a short-term wind power prediction program to forecast the next day grid constraints and congestions. These predictions have a lower accuracy than load forecasts, and this uncertainty should be also considered, for instance, for the calculation of the available interconnection capacity between systems.

The paper is structured as follows: first, a short view of short-term wind power prediction, and the uncertainty associated to its output, is given. Then, the linearization process is described in detail, including a distributed slack bus procedure. Next, the theoretical foundations of the linearization process are explained and justified, together with the Cornish–Fisher expansion. The method is applied to a test grid, and its results are compared to Monte Carlo simulation and point estimate method, and commented. A summary with the main conclusions follows, and appendices with further explanations end the paper.

## 2. Short-term wind power prediction: uncertainty

### 2.1. Short-term wind power prediction

Short-term wind power prediction programs are tools that provide an estimation of the future power production of a wind farm, or a group of wind farms, in the next hours. For this purpose, they use meteorological forecasts coming from a numerical weather prediction (NWP) tool, and sometimes real time SCADA data from the wind farms, as wind power production, measured wind speed, etc. Data of the wind farms, such as rated power, type and availability of wind turbines, etc. are also necessary. The output of these programs is the hourly average wind farm production for the next hours. Typically, predictions are issued for the next 48 h, but longer time horizons are possible, sometimes at the price of a lower accuracy.

These predictions tools are less accurate than load prediction programs and their accuracy decreases with the time horizon. A survey of the accuracy of these tools is given in [10], and an example for a typical wind farm, where the output from the prediction program SIPREOLICO, compared to *persistence* is shown in Fig. 1. Persistence is a prediction method that assumes that the future production, for the entire time horizon considered, is the current production of the wind farm, i.e., $p^*(t+k|t)=p(t)$, where $p^*(t+k|t)$ is the power predicted at time $t$ for $k$ hours later, and $p(t)$ is the power generated by the wind farm at time $t$. This method is considered as a threshold of the performance of a forecasting method. SIPREOLICO is a prediction program used since year 2002 in Red Eléctrica de España, the Spanish TSO, and developed by Universidad Carlos III de Madrid, to forecast wind power production of the next 42 h, every 15 min, for the 14 GW of wind power connected to the Spanish peninsular grid (end 2007). Details of SIPREOLICO tool can be found in [9].

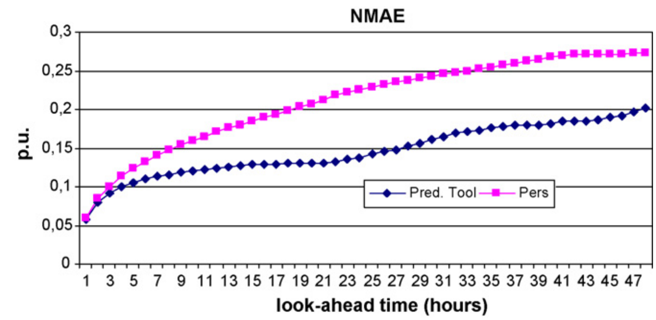


**Fig. 1.** . NMAE of SIPREOLICO and persistence for a typical wind farm.

Fig. 1 represents the Normalized Mean Average Error, defined as

$$NMAE(k) = \frac{1}{P_n} \frac{\sum_{t=1}^{N} |e(t+k/t)|}{N} \quad (1)$$

where

$$e(t+k/t) = p(t+k) - p^*(t+k/t) \quad (2)$$

and $p(t+k)$ is the production of the wind farm at time $(t+k)$. $P_n$ is the nominal power of the wind farm, and $N$ is the number of predictions examined along the considered time. It can be seen that the wind power prediction accuracy allows for much uncertainty, and that the actual value may differ widely from the predicted one.

### 2.2. Uncertainty of short-term wind power prediction

The predictions provided by a short-term wind power prediction program are uncertain, and this uncertainty must be modeled for an adequate assessment of these predictions.

Let $p$ be the random variable associated with the power output of a wind farm. Then, the probability of producing $p$ MW, having predicted a value of $p^*$ MW $k$ hours before, is given by the probability density function $f_{p^*,k}(p)$.

The uncertainty, and hence the probability density function, changes with the range of the wind farm power output, since this value is bounded between zero and the rated power. Besides, the power curve of a wind turbine or wind farm is non-linear. If we assume that the wind speed predictions have Gaussian uncertainty, then the probability density functions of the power predictions will not be Gaussian [10]. The shape of these probability density functions is also affected by the time lag elapsed between the prediction and the operation times. Predictions with a shorter time lag are more accurate, and their variance is expected to be smaller than those predictions produced longer before. To obtain analytically, or in real time, the uncertainty of this prediction is difficult, but approximate estimates can be made from past data, and some research has already been made in this field [10].

It is not the purpose of this paper to propose a model for this uncertainty, and a reasonable assumption will be used as an approximation. Due to the bounded nature of the power produced by a wind farm, a Beta PDF will be used, as proposed in [11]. Heuristic PDF, as shown in [12], supports this assumption, although this is still an open field for research. Appendix A gives the analytical expression of Beta distribution. Other PDF, even non-parametric approaches, could also be considered.