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New paradigms towards the modelling of complex systems in behavioral economics

G. Ajmone Marsan *

IMT- Institute for Advanced Studies, Piazza S. Ponziano, 6, 55100, Lucca, Italy
EHESS, 54, boulevard Raspail 75 006, Paris, France

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ABSTRACT

This paper develops a mathematical framework based on kinetic theory for active particles and on a suitable decomposition into functional subsystems and shows how it can be implemented to describe some specific complex economic applications. Specifically, the applications are focused on opinion dynamics and job mobility phenomena. These two examples offer a first insight into multiscale issues: starting from the application, a preliminary mathematical framework taking into account both microscopic and macroscopic interactions is developed. This framework may be adapted to the modelling of a great variety of complex phenomena.

1. Plan of the paper

The development of a mathematical description of complex socio-economic systems is a fascinating, however difficult, perspective of interaction between mathematical and social sciences. An important objective consists in the assessment of the paradigms which can act as a conceptual background for a unified approach to the modelling of a variety of social systems characterized by common features.

This program is pursued by suitable developments of the so-called mathematical kinetic theory for active particles KTAP [1,2], which has been shown to be a useful reference application in various fields of life sciences such as the immune competition [3], traffic flow [4], social dynamics [5,6], psychological interactions [7,8].

A modelling approach to complex socio-economic systems has been recently developed using the mathematical tools offered by the KTAP theory [9,10]. These papers offer various hints to revisit this theory by introducing the concept of stochastic games and network structure together with the multiscale approach already introduced in [11]. In particular, multiscale aspects are an inner feature of all complex systems in general and of complex social systems in particular. It is worth remarking that the common features of complex systems allow us to update mathematical tools in different fields of life sciences and transferring them, after suitable developments, from one field to the other.

Several attempts to combine social sciences and mathematical modelling can be found in the literature: in particular the main branches are agent-based models, game theory, population dynamics, and social networks.

Agent-based models are computational models in which interactions among individuals (agents) are simulated, in order to understand the emerging behavior of the macroscopic overall system, starting from the microscopic dynamics of each agent. The goal is to find the equilibria or the emergent patterns in the complex system, as a consequence of the dynamics of the agents. This approach combines elements of game theory, multiagent systems and Monte Carlo methods. These models

* Corresponding address: IMT- Institute for Advanced Studies, Piazza S. Ponziano, 6, 55100, Lucca, Italy.
E-mail address: g.ajmonemarsan@imtlucca.it.

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consist in systems of equations that simulate the simultaneous behavior of multiple agents, starting from specific sociological assumptions, in order to recreate and predict the global behavior of the system.

Agent-based models can be considered as a complement to traditional analytic methods, since they may recreate the equilibria or non-equilibria resulting from the analytic method itself. Indeed, they can explain the emergence of unpredictable patterns and distinguish among different types of network structures. These models have been recently used to describe a great variety of network-related phenomena of recent times, like internet, terrorism, traffic jams, financial crisis, consumer behavior, spread of epidemics and social segregation. Notice that all the phenomena listed above are strongly affected by individual or particle interaction and behavior. Some key references can be found in [12,13].

Game theory is a branch of applied mathematics that has become widely used in social sciences and economics. The goal of game theory is to describe the behaviors of individuals (players), who have to design specific strategies to give the best response to other players' choices. Game theory is widely applied in all the rational modelling of social sciences. It aims at finding equilibria in the sets of strategies and different kinds of equilibria have been defined. Starting from the analysis of economic competition with the 1944 book *Theory of Games and Economic Behavior* by John von Neumann and Oskar Morgenstern [14], game theory is now widely used to model biological, social and even telecommunication interactions. Most notable references can be found in [15–17].

Population dynamics, instead, studies how the number of individuals in one or several populations changes, under the action of biological and environmental processes. It focuses on the rise and decline of different populations of different particles: individuals, cells and so on. It has been a dominant branch of mathematical applications to biology for more that 200 years. In the early 19th century it widely investigated demographic studies, whose main modelling aspects are related to the following dynamic rate functions of the considered populations: natality or birth rate, growth rate and mortality. For a complete survey on the subject it is possible to refer to [18].

Particularly important is, in economic and social applications, the use of population dynamics with internal structure, where an additional internal variable can model specific characteristics of the system under consideration. Large systems of interacting individuals can be modelled by systems of partial differential equations taking into account internal variables describing the socio-economic state of the individuals. This internal structure describes the peculiarities of the system under consideration, and contributes to model the emergence of a collective behavior. An introduction to population dynamics with internal structure is given in [19–21].

Notice that, in population dynamics, the internal structure is a deterministic variable, in the KTAP, instead, the internal variable is heterogeneously distributed among agents, while interactions modify such distribution. Indeed, this is an essential ingredient to model social systems in the framework of behavioral economics.

The study of *social networks* is receiving growing attention, focused on the role of networks in determining and constraining social behavior. Starting from Milgram's experiment [22], and the celebrated paper developing the concept of *small-world phenomena* [23], it is possible to find, in the recent literature, many socio-economic phenomena studied by this approach [24–26]. In particular, [27,28] show the importance of social networks and interactions in economics. The wholesale fish market in Marseille has been studied in [27], introducing the idea of high loyalty of buyers to sellers, and persistent price dispersion taking place in the same place every day. The authors build an adaptive agent model, where sellers set quantities, prices and treatment to customers, buyers instead decide which seller to visit and at which price to buy. Subsequently, the emerging behavior is analyzed. The second work [28], instead, is a collection of articles having in common the mathematical modelling approach to economics, which takes into account *heterogeneous interacting agents*. In its introduction, the reconciliation among economics, psychology and sociology, is highlighted thanks to the new cognitive approach towards economics in general.

The borders among these disciplines show, in practical applications, various flexibilities, in the sense that tools and methods of each of them are used together for a better development of mathematical models. In particular, a recent field of investigation, which wisely combines elements of the three branches described above is the so-called *behavioral economics*. Its principal scope is to merge properly psychology and economics, stressing the importance of human behavior and psychological attitude in the evolution of complex systems of individuals. Behavioral economics widely use experiments as a preliminary step of the modelling process. To obtain an idea of the wide applications and philosophical foundations of behavioral economics it is possible to refer to [29–39]. Moreover [40,41] provide an insight into the links between behavioral economics and game theory, in order to adapt the second to the challenges proposed by the first, introducing the concept of *behavioral game theory*. Additional references can be found in [42,43].

It is plain that stochastic aspects should characterize models derived within the framework of the behavioral economy, consequently models derived by KTAP methods, seem to be appropriate to investigate this kind of phenomena. This theory can be briefly summarized as follows. Social systems are characterized by a large number of particles representing the interacting socio-economic entities, called active particles, (consumers, firms, institutions and so on), whose state may be characterized by both mechanical variables and an activity variable representing the specific socio-economic feature of the system itself. While the activity variable describes the microscopic state of the system, the global state is described by the probability distribution function over the microscopic state whose weighted moments recover the macroscopic quantities of the system. Notice that thanks to the activity variable, the output of the interactions is a direct consequence of the strategic behavior which takes place among active particles. The activity variable, in the more general case, can be different from particle to particle. The evolution in time and space of the system is derived by a balance of the inflow and outflow of active particles in the elementary volume of the space, as a consequence of the dynamics based on strategic interactions.