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Games with strategic complementarities: New applications to industrial organization☆

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Abstract

This paper provides an introduction to the analysis of games with strategic complementarities and applications to industrial organization: oligopoly pricing, comparative statics and a taxonomy of strategic behavior in two-stage games.

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1. Introduction

Games of strategic complementarities are those in which the best response of any player is increasing in actions of the rivals. Many games usually studied in industrial organization display strategic complementarities including a large subset of those involving search, network externalities, oligopoly interaction, or patent races. Recently, there has been a surge of interest in the study of competition in the presence of complementarities in industries with a network component, such as credit cards, or where systems competition is important, like in software.

Supermodular games (Topkis, 1979; Vives, 1985a, 1990; Milgrom and Roberts, 1990) provide the appropriate framework to model strategic interaction in the

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presence of complementarities. The theory of supermodular games is based on a lattice-theoretic approach that exploits order and monotonicity properties. Both existence of equilibrium and comparative static properties are based on order and monotonicity properties in contrast to the usual box of tools based on convex analysis and calculus.

The approach is powerful and delivers strong results. In the class of supermodular games the existence of equilibrium in pure strategies is ensured without requiring quasiconcavity of payoffs; the equilibrium set has an order structure, having extremal elements that allows a global analysis of the set; there is an algorithm to compute extremal equilibria, which also bound the rationalizable set; and monotone comparative statics results are obtained with minimal assumptions.

The purpose of the paper is to provide an introduction to the class of supermodular games and provide some applications in industrial organization analysis. We obtain new results and new light is cast on old results by getting rid of unnecessary assumptions. In this way the role of the critical assumptions is highlighted. At the same time the range of application of the theory is extended beyond games of strategic complementarities, providing examples of results obtained in games displaying strategic substitutability. The reader is warned however that the approach, although useful in a very large class of cases, is not of universal applicability.

The plan of the paper is the following. Section 2 presents the basic results of the theory and some examples. Section 3 develops some applications to oligopoly pricing in homogenous and differentiated products environments as well as comparative static results. Section 4 extends the taxonomy of strategic behavior due to Fudenberg and Tirole (1984). Concluding remarks end the paper.

2. An introduction to games with strategic complementarities

This section contains basic definitions and some of the main results in the theory of supermodular games. The reader is referred to Vives (1999, in press) for a more thorough and general treatment of the theory, as well as proofs, and further references and applications.

The definition of a game with strategic complementarities is provided in a smooth context. This is done only to minimize the mathematical apparatus but it is not the most general way to define it. A game $(A_i, \pi_i; i \in N)$ is defined by the set of players N, $i=1, \ldots, n$, by the strategy set A_i and the payoff π_i of player $i \in N$ (the payoff is defined on the cross product of the strategy spaces of the players A). Let $a_i \in A_i$ and denote by a_{-i} the strategy profile $(a_1, \ldots, a_n)$ except the ith element. We have then $a_{-i} \in \Pi_{j \neq i} A_j$. The game $(A_i, \pi_i; i \in N)$ is *smooth supermodular* if each A_i is a compact cube in Euclidean space, and $\pi_i(a_i, a_{-i})$ is twice continuously differentiable with

(i) $\partial^2 \pi_i / \partial a_{ih} \partial a_{ik} \geq 0$ for all $k \neq h$ and
(ii) $\partial^2 \pi_i / \partial a_{ih} \partial a_{jk} \geq 0$ for all $j \neq i$ and for all h and k,

where a_{ih} denotes the hth component of the strategy a_i of player i.