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# The derivation and calibration of the life-quality index (LQI) from economic principles

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**Abstract**

The life-quality index (LQI) is a versatile tool to support the effective implementation of programs and practices for managing risk to life safety. The LQI allows a transparent and consistent basis for determination of the net benefit arising from projects, programs, standards and policies undertaken at some cost to improve safety or enhance the quality of life. The paper shows that the LQI model is in harmony with well-established principles of economics, utility theory and recent developments to quantify the progress of nations through indicators of human development. The initial calibration of the LQI was based on a simplifying assumption of a linear relation between the GDP and work time. In this paper, we modify the calibration using empirical data for GDP and work time and link the LQI model to well-established economic principles and theory of production. The proposed improvements to the model eliminate a systematic bias associated with estimation of societal willingness to pay for safety. In addition, it provides a rigorous basis for program evaluation to assist decision-makers in directing expenditures where they may most effective.


*Keywords:* Life-quality index; Risk management; Cost-benefit analysis; Utility function; Macroeconomics; Gross-domestic product; Cobb–Douglas production function

## 1. Introduction

### 1.1. Background

Efficient management of risks to life safety involves the search for a balance between the overall potential for harm and good outcomes. In recent years several acceptability criteria with quantitative rationale have been derived from compound social indicators ([12,15,10,23]) to support evaluation of broad program outcomes. A prominent and recent example of this is the Human Development Index (HDI). Developed under the auspices of the United Nations Development Program to compare internationally the level of social

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development of nations. The HDI combines three social indicators, the gross domestic product per person per year (GDP), life expectancy at birth (LE) and education, and ranks nations accordingly. The UNDP approach does not provide any guidance about how such an index could be used to help guide practical decisions around funding or programs and choices.

Lind et al. [12] recognized that risk management is not only about engineering and economic efficiency of investment but, more importantly, it is about improving the overall public welfare by reducing risk to life in a cost-effective manner. They proposed the use of two key social indicators, real GDP per capita and life expectancy (LE) (already identified in the UN Human Development Project) for judging the effectiveness of decisions about risk and life safety. The concept was expanded by Nathwani et al. [15] who further developed the life-quality index (LQI) to establish a test of efficiency for programs and regulations to manage risks.

Using a social indicator to derive an objective value of acceptable risk places risk mitigation in the context of national goals implicit in the indicator. The HDI aims to reflect how well a nation enables its citizens to live long, healthy and enriched lives. The HDI reflects a set of values that focuses on human development and measures progress of nations in achieving them through fine tuning the GDP, corrected for purchasing power parity (Lind, 2004). The HDI and the LQI rank developed nations quite similarly (Lind, 2003).

The LQI is simpler and based on better-defined component indices weighted to reflect peoples' revealed preference for the work/non-work time ratio and productivity. It allows an explicit valuation of a project's effectiveness for life extension.

Although the use of social indicators to track progress of nations is a recent development, the philosophical foundations of welfare economics was established earlier by Pigou [22] and Hicks [4,5], and these ideas continue to influence the development of social and economic policies centered around the concept of human welfare. The LQI is an innovation that builds on the concept of a social indicator that is a function of mortality and economic production and places an implicit value on reduction of life risk. The implied value is the increase in wealth production (i.e., real GDP per person) required to neutralize a small unit increase in mortality ([12,15,10,23], Lind, 2003, 2004). Use of LQI based on key social indicators offers the great advantage in that it provides a criterion of acceptable risk that harmonizes with national social and development objectives reflected in the social indicator.

The basis for the derivation and calibration of the LQI parameters has engendered discussion in the analytical community and further elaboration of the LQI framework will assist practitioners in implanting the concept in decision making. The purpose of the paper is to present an analytical approach to the derivation and calibration that is consistent with established principles of economic sciences. The proposed LQI calibration is based on the concepts of production economics utilizing available economic data, thereby, removing a simplifying assumption used in the original study.

### 1.2. The life-quality index

Originally the LQI was presented as a function of the real GDP ($G$ $/person/year) and life expectancy ($E$ years/person) [15] as follows:

$$L_{\mathrm{O}} = G^{c}E^{(1-c)}, \tag{1}$$

where $c$ is a constant, denoting the annual fraction of work time per person required for producing $G$. The LQI was derived on the basis of a differential equation of LQI with some restrictions placed on its coefficients. The differential equation approach is considered less intuitive and there is a need to provide a fuller explanation of for the wider use by decision-makers.

Subsequently, Pandey and Nathwani [18–20] presented a derivation of LQI using the concept of a lifetime utility function and improved its technical foundation. In this formulation, the LQI turns out to be $(1-c)$th root of the original index given in Eq. (1):

$$L_{\mathrm{Q}} = G^{c/(1-c)}E \quad (= L_{\mathrm{O}}^{1/(1-c)}). \tag{2}$$

This derivation provides a richer explanation of the underlying concepts and it links the LQI to concepts generally understood by practitioners in decision analysis, economic modelling, cost-benefit analysis and risk assessment.