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Prediction of the learning curve in roof insulation

Levente Mályusz *, Attila Pém

Budapest University of Technology and Economics, Faculty of Architecture, Hungary

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ABSTRACT

Mathematical learning curve models can be used in construction to predict the time or cost required to perform a repetitive activity. In this study, we evaluated mathematical models for different learning curves for flat roof insulation reconstruction work. Our evaluation was based on a survey conducted in the spring of 2009 in Budapest. The survey was conducted to determine the total construction time required for bound parts of flat roofs and for the related activities, such as demolition or laying heat insulation boards. Several mathematical models were identified, and each was used for prediction. The objective of this study was to determine which of the models considered was the most accurate for the prediction of future performance. The models were compared with each other and with the measured data.

1. Introduction

The aim of this paper is to describe the result of an exploratory study to evaluate the predictive capabilities of various learning curve models—mathematical models and data presentation methods—using them for roof insulation work and to compare data from field studies with those reported in the literature. The data for this study were collected by writers in a real reconstruction work of flat roofing insulation work. Time to complete 1 cycle was measured only. The workers were not disturbed. The timer was stopped when workers took a break. Our expected result was to find a mathematical model and a data presentation method to predict future performance of activities for roof insulation works.

The basic principles of learning curves are well understood. Learning curves imply that when numerous similar or nearly identical tasks are performed, the effort is reduced with each successive task (Fabrycky et al. [1], Ostwald [2], Oglesby et al. [3], Drewin [4], Teplitz [5], Everett and Farghal [6,7], Lutz et al. [8], Lam et al. [9], Couto and Teixeira [10]). Learning curve theory can be applied to predicting the cost and time, generally in units of time, to complete repetitive activities. The cumulative average time is the average time required to perform a given number of activities. The cumulative average time method was used in the original formulation of the learning curve method, referred to as Wright's model, in Wright's famous paper on the subject [11]. In Thomas et al. [12] it is concluded that the best predictor is a cubic model. A number of researchers have suggested that Wright's model is the best model available for describing the future performance of repetitive work (Everett and Farghal [6], Couto and Teixeira [9], Wong et al. [13]). A few construction companies use learning curve computations of in-place construction costs on on-going construction jobs to make projections of the costs and time of future work to be performed. There is little information in the literature about these uses, although it seems that the learning curve principle can be applied to repetitive construction operations (Hinze and Olbina [14]). In this study, we evaluated mathematical models for learning curves based on Everett and Farghal [6] and investigated data presentation models based on Everett and Farghal [7] and Mályusz and Pém [15].

2. Mathematical models and methods

2.1. Mathematical models

Learning curve theory is applicable to the prediction of the cost or time of future work, assuming repetitive work cycles with the same or similar working conditions in terms of technology, weather, and workers, without delay between two consecutive activities. The direct labor required to produce the (x + 1)st unit is assumed to always be less than the direct labor required for the xth unit. The reduction in time is a monotonically decreasing function, an exponential curve, as described in Wright's [11] paper.

In this study, we calculate the labor hours/square meter for each repeated activity.

Wright's linear log x, log y model is as follows:

$$\ln y = \ln a + b \ln x; \quad \text{or} \quad y = ax^b = ax^{\log_2 r} \tag{1a}$$

where x is the cycle number, y is the time required to complete cycle x in labor hours/square meter, a is the time required to complete the first cycle, b is a learning coefficient, and r is the rate of learning. For example if $r = 0.9$ (90%), then $b = -0.151$. Wright discovered that when the labor cost decreases at a constant rate, that is, the learning rate, the production/cycles doubles. So learning rate is the constant

* Corresponding author.
E-mail addresses: lmalyusz@ekt.bme.hu (L. Mályusz), apem@ekt.bme.hu (A. Pém).

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rate with which labor time/cost decreases when the production/cycles doubles in a linear log x, log y model. This feature of the learning rate comes from the logarithms nature and true only in linear log x, log y model. We do not define the rate of learning in the other models.

The mathematical models evaluated in this survey are shown as Eqs. (1a), (1b), and (1c).

Linear x, log y model:

$$\ln y = \ln a + x \ln b; \quad \text{or} \quad y = ab^x. \tag{1b}$$

Linear log x, y model:

$$y = \ln a + b \ln x; \quad \text{or} \quad \exp(y) = ax^b \tag{1c}$$

where x is the cycle number, y is the time required to complete cycle x, and a and b are model parameters.

In this study, we investigated the application of learning curves to flat roof insulation reconstruction work. The objective of this study was to examine different methods of representing learning curve data and investigate which method can be used to obtain the most accurate approximation. Unit, cumulative average (CA), moving average (MA) and exponentially weighted average values with $\alpha = 0.3$ (EA(0.3)) and $\alpha = 0.5$ (EA(0.5)) are presented to predict the time required for the insulation work. The linear log x, y model is applicable only to a small number of repetitive items because y is a decreasing function only if $b < 0$, but in this case, $y \to -\infty$.

2.2. Data presentation

The unit is the data item that represents the time required to perform 1 cycle of the insulation work.

The cumulative average was defined by Wright [11]. He discovered that the cumulative average (CA) time decreased by a fixed percent when the output doubles. CA represents the average time or cost of different quantities (X) of units.

$$CA_t = (Y_1 + Y_2 + \ldots Y_{t-1} + \ldots + Y_t)/t. \tag{2}$$

where t is the number of cycles, CA_t is the cumulative average in cycle t, and Y_t is the unit datum for cycle t.

The moving average (MA) in this paper is the average time of the last 3 cycles. Although the MA is an average like the CA, the MA represents the most recent data. The analyst can decide how far back in time the data are still relevant. More points will help smooth the curve. In extreme cases, moving averages are unit data or cumulative averages.

$$MA_t = (Y_t + Y_{t-1} + Y_{t-2})/3 \tag{3}$$

The exponential average (EA) is a weighted average of the most recent data and the previous average.

$$EA_t = \alpha Y_t + (1-\alpha)EA_{t-1} \tag{4}$$

$$EA_{t-1} = \alpha Y_{t-1} + (1-\alpha)EA_{t-2} \tag{5}$$

$$EA_{t-2} = \alpha Y_{t-2} + (1-\alpha)EA_{t-3} \tag{6}$$

That is,

$$EA_t = \alpha Y_t + \alpha(1-\alpha)Y_{t-1} + \alpha(1-\alpha)^2 Y_{t-2} + (1-\alpha)^3 EA_{t-3} \tag{7}$$

where EA_t is the exponential average time for cycle t, EA_{t-1} is the exponential average time for cycle $t - 1$, Y_t is the unit data (time to perform an activity) in cycle t, and α is a coefficient. If α is greater than 0.5, then the effect of new data is greater than that of older data. In this study, two values of α, 0.3 and 0.5, were examined, based on Everett and Farghal [7].

Our assumption is that an exponential relationship exists between Y_t and x, i.e., between the time required to complete the activity for a given cycle and the cycle number. In other words, our assumption is that Eq. (1a) holds.

The relationship between log Y_t and log x described by Eq. (1a) can be plotted as a straight line on log–log paper, and all the regression formulae apply to this equation just as they do to the equation. Mathematically, the latter is solvable for parameters a and b using the least squares method. In the case of Eq. (1b), there is also a linear relationship between x and log Y_t, so the values of the parameters a and b can be calculated using the least squares method. In Eq. (1c), there is a linear connection between log x and Y_t, so the values of the parameters a and b are calculated as in the previous case.

3. Analyzing the methods by application to a real construction project

3.1. Description of the project

The data for this study were collected by writers in a real roof insulation work. The surveyed project was a reconstruction of flat roofing. During the reconstruction process, the circumstances and the weather were ideal for roofing (sunny, 26–33 °C, no wind). The same workers performed the entire project. The technology was repetitive within one part. The workers knew that they were being monitored, but they were not informed as to what was being measured, and they were not disturbed.

In the part of the reconstruction process that was studied, the work under consideration consisted of the following activities: slicing up the old waterproofing, laying down 10-cm-thick heating insulation and attaching it to the roof using screws, spreading one layer of EPDM (Ethylene Propylene Diene Monomer) rubber waterproofing, and melting it to the cape of the screws. The joining, the fixing of the edges, and the changing of the roof windows were not surveyed.

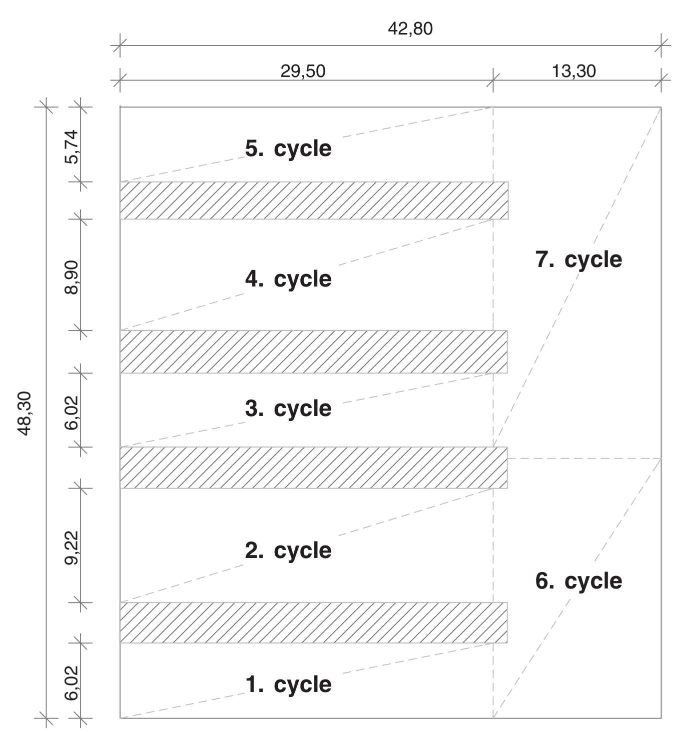

Fig. 1. Hall building roof.