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Intra-layer network coding for lossy communication of progressive codes

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ABSTRACT

In this paper we discuss layered multicast (LM) of progressive source codes using network coding. LM is absolutely optimal if different sinks in the network are satisfied up to their max-flow. Since absolutely optimal intra-layer network strategies might not exist for general networks, we present conditions under which an absolutely optimal, intra-layer multicast strategy exists for a given network and how that strategy may be efficiently constructed. We also discuss the problem of designing optimal intra-layer multicast strategies for general directed networks.

1. Introduction

Multiple description codes can effectively utilize network resources and is able to combat packet loss in communication networks [1]. The network communication throughput can be improved with network coding as compared to routing only strategies [2,3]. The problem of lossy source communication using network coding becomes very complex in its most general form [3–5]. A practical subclass of the problem uses progressive source codes along with carefully optimized network coding strategies for efficient multicast of compressible sources [6,5], and is the subject of this paper.

We start by introducing a generalization of network coding problem, called Rainbow Network Coding (RNC) [7]. RNC recognizes the fact that the information communicated to different members of a multicast group can be different in general. Take the scenario depicted in Fig. 1 as an example. Here, two information bits are to be communicated from node 1 (source) to the sink nodes. The capacity of all links is one. First, lets assume that nodes 4, 5 are sinks. By network coding theorem [2], two bits a, b can be communicated to nodes 5, 6 simultaneously as indicated in Fig. 1(a). Here, transcoding at relay nodes plays a crucial role. In particular, node 7 combines the two bits it receives to produce $a \oplus b$ (where $\oplus$ indicates XOR operation) which enables nodes 4, 5 to recover both bits. It is easy to verify that at most 3 bits can be communicated to the nodes 4, 5 using routing only. Therefore, the multicast throughput with routing only is at most 1.5 bits per second per node, when the set of sink nodes is $T = \{4, 5\}$ as shown in Fig. 1(b).

Now, what if the set of sink nodes is $T = \{4, 5, 6\}$? Since the min-cut (and hence max-flow) into node 6 is only one, the multicast capacity of the network with sinks $\{4, 5, 6\}$ is only one. Therefore, only one bit of *common information* per second can be communicated to each sink. Both examples in Fig. 1, however, are able to communicate a total of 4 information bits to nodes 4, 5, 6. In Fig. 1(a), nodes 4, 5 each receive 2 bits, while node 6 does not receive any (a total of 4 bits). In contrast, in Fig. 1(b), nodes 4, 6 each receive one bit, while node 5 receives two bits (again, a total of 4 bits). Note that in this case, node 4 is not satisfied up to its max-flow. We can clearly see from this example that all the three nodes in T cannot be satisfied up to their max-flow and hence an absolutely optimal LM strategy cannot be constructed.

If each atomic data entity for transmission is assigned a unique "color", the above problem is about delivering to each node with as large a "spectrum" of colors as possible, without requiring all sink nodes to have exactly the same set of colors. Due to this analogy, we call this form of network information flow problem RNC, a generalization of rainbow network flow (RNF) to the case where network coding is allowed [8].

Both examples in Fig. 1 are valid rainbow network codes. In Fig. 1(a), the RNC involves transcoding of information at the relay node 7 (the XOR operation). The example in Fig. 1(b), however, uses routing only. This subclass of RNC is called RNF [8].

In this paper, we discuss the idea of layered multicast (LM) [6,5]. In LM, progressive or layered source codes are used for source compression, while network coding is used to multicast these coding layers [3]. The receivers that receive more layers are able to reconstruct the source at lower distortions. The multicast

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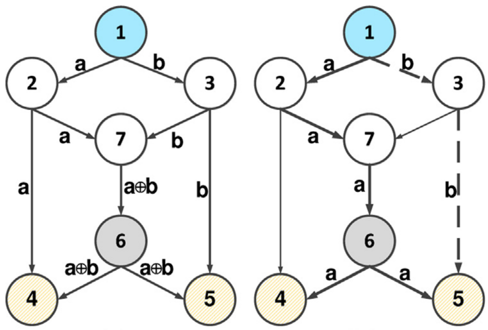

Fig. 1. Multicasting common information: (a) using network coding two bits of information can be sent to nodes 4, 5 (b) with routing only, however, two bits can be sent to node 5, one bits to each of the two nodes 4, 6.

strategies, however, should respect the layered structure of the codes, i.e., a layer should be delivered to a receiver, only if the receiver has received all the previous layers [9]. Such restricted reception mechanisms, however, do not lead to accurate algorithm models that provide the absolute optimal performance [3]. The reason is that nodes at critical network locations may need to assist in forwarding of data just for the benefit of downstream peers.

We formulate the problem of LM and define important terms in Section 2. Since absolutely optimal intra-layer network strategies might not exist for general networks, we discuss conditions under which an absolutely optimal, intra-layer multicast strategy exists for a given network and how that strategy may be efficiently constructed in Section 3. The problem of designing optimal intra-layer multicast strategies for general directed networks is investigated in Section 4 and the paper is finally concluded in Section 5.

2. Formulation

The problem of LM is defined on a directed acyclic multi-graph $\mathcal{G}\langle V, E\rangle$, a single server node $s \in V$, and a set of clients $T \subset V$. Each network link e has capacity $C(e)$ that indicates the average number of bits that can be communicated over e without error, per network use. For any integer n, the multi-graph $G_n\langle V, E\rangle$ is created by replacing each link of $\mathcal{G}\langle V, E\rangle$ with $\lfloor n \times C(e) \rfloor$ links, each with capacity 1. A total of K information messages called *layers* are generated at s of lengths $h_1 = n \cdot r_1$, $h_2 = n \cdot r_2$, ..., $h_K = n \cdot r_K$ for some $r_k \in \mathbb{R}^+$, $k = 1, 2, \ldots, K$. We will assume a fixed large integer n is given such that h_k's can be approximated by integers, and we deal with $G_n\langle V, E\rangle$ only. The dependance of the parameters on the block size n is understood throughout. The use of integer link capacity is a well known technique to simplify graph theoretical arguments and does not restrict the generality of the derivations.

Definition 2.1 *(Multicast session and multicast strategy).* A multicast session is identified by a tuple $\Gamma_k(h_k) = (G_k, h_k)$, where G_k is a subnetwork (not necessarily a sub-tree) of G. A multicast strategy Γ is an ordered collection of K multicast sessions, $\Gamma = (\Gamma_1(h_1), \Gamma_2(h_2), \ldots, \Gamma_K(h_K))$, such that each edge e belongs to at most one multicast session.

Multicast session k is used to multicast layer k of length h_k bits. Define $V_k \subset V$ as the set of all nodes in G_k that are able to recover all the h_k bits in layer k without error.

Definition 2.2 *(Subscription).* A client node t *may subscribe* to a multicast session k if $t \in V_k$. Define $T_k \subset V_k \cap T$ as the set of all clients that subscribe to layer k. A subscription strategy Γ is said to be layered if and only if for all T_k, $k = 2, 3, \ldots, K$ one has $T_k \subset T_{k-1}$.

Definition 2.3 *(Layered flow vector).* The total number of bits of layered messages received by node t is defined as $q_t = \sum_{k: t \in T_k} h_k$. The vector $(q_t\,; t \in T)$ is called the *layered flow vector*.

Definition 2.4 *(Achievable layered flow vector).* Define $Q \in \mathbb{R}^{+|T|}$ as the union of all layered flow vectors $(q_t\,; t \in T)$ for all layered multicast strategies Γ.

Let $(f_t\,; t \in T)$ be the value of max-flow to the sink nodes in T. The maximum number of information bits that can flow from s to t is at most f_t bits per network use [2].

Definition 2.5 *(Inter-layer vs. intra-layer network coding).* If network coding is applied to the communication of each session separately, the layered coding strategy is called "intra-layer" coding. On the other hand, if the layers in different sessions are encoded together, the layered coding strategy is called "inter-layer" coding.

Note that the set of achievable flow vectors using intra-layer network coding is a subset of that achievable with inter-layer network coding [3–5,10].

Definition 2.6 *(Absolute optimality).* Since f_t is the max-flow into t, we must have $q_t \le f_t$ for all $t \in T$. If $(f_t\,; t \in T) \in Q$, i.e., if there exists a layered multicast strategy that can deliver the maximum possible flow to each client, we say that layered multicast is absolutely optimal and the corresponding layered multicast strategy is called absolutely optimal layered multicast strategy.

Remark 2.1. If $f_{\min} = \min_{t \in T} f_t$ then $(f_{min}\,; t \in T) \in Q$, i.e., a flow vector where all components are equal to f_{min} is always achievable. In particular, an absolutely optimal layered multicast strategy with one multicast session always exists if all the sink nodes have the same max-flow.

3. Conditions for absolute optimality of LM with intra-layer network coding

In multi-rate multicast problems where clients have different max-flows [6], absolutely optimal multicast strategies, as defined in Section 2, may not exist. We start by conditions under which an absolutely optimal, intra-layer multicast strategy exists for a given network and how that strategy may be efficiently constructed. We then move to optimization strategies for intra-layer coding in general networks. The following theorems show the special cases where LM with intra-layer network coding is absolutely optimal.

Theorem 3.1. *An absolutely optimal LM strategy with at most two layers always exists if the number of sink nodes is at most two. Counter examples with three clients can be found for which no absolutely optimal LM strategy exists.*

3.1. Proof of Theorem 3.1

If the sink nodes have the same max-flow (which includes the case with only one sink as trivial special case), the absolutely optimal LM strategy has a single multicast session which consists of the set of maximum edge disjoint paths to the receivers. We only need to prove the theorem for the case of two sink nodes with max-flows $f_1 < f_2$ without loss of generality. We know that there are a maximum of f_i edge disjoint paths to the sink nodes i, for $i = 1, 2$. Let $A = \{a_1, a_2, \ldots, a_{f_1}\}$ and $B = \{b_1, b_2, \ldots, b_{f_2}\}$ be two sets of maximum edge disjoint paths into the sink nodes 1 and 2, respectively. Define $B_\cap = \{b_1, b_2, \ldots, b_{f_1}\}$ and $B_* = \{b_{f_1+1}, b_{f_1+2}, \ldots, b_{f_2}\}$.

Suppose for a moment that the f_1 paths in A are edge-disjoint from the paths in B_*. In this case, we can construct the multicast strategy with two multicast sessions as follows: the first layer consists of the union of paths in $B_\cap$ and A, i.e., $\Gamma_1 = B_\cap \bigcup A$ of size f_1. The second layer is $\Gamma_2 = B_*$ and has rate $f_2 - f_1$. Node 2 subscribes to