Signal Processing 98 (2014) 212–223

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Unbiased consensus in wireless networks via collisional random broadcast and its application on distributed optimization

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ARTICLE INFO

Article history:
Received 9 May 2013
Received in revised form
26 September 2013
Accepted 12 November 2013
Available online 1 December 2013

Keywords:
Consensus
Random broadcast
Gossip
Distributed optimization

ABSTRACT

We first propose an unbiased consensus algorithm in wireless networks via random broadcast, by which all the nodes tend to the initial average in mean almost surely. The innovation of the algorithm lies in that it can work in any connected topology, in spite of the possible collisions from simultaneous data arriving at receivers in a shared channel. Based on the consensus algorithm, we propose a distributed optimization algorithm for a sum of convex objective functions, which is the fundamental model for many applications on signal processing in network. Simulation results show that our algorithms provide an appealing performance with lower communicational complexity compared with existing algorithms.

1. Introduction

In a multi-hop wireless network, each node communicates with others via broadcast wireless media, where data are usually exchanged among neighboring nodes due to the power constraint. In the early period of the study on wireless networks, many researchers focused on the multi-hop communication strategies, such as the MAC and routing protocols [1]. With the development of sensor network and Internet of Things (IoT), a smart network embraces necessary signal processing jobs in-network [2], which considers the data transmission and processing in integrated way. The potential applications include distributed estimation [3–6], data fusion [7], classification [8], and recovery for sparse signals through l_1-norm minimization [9].

Consensus is a representative topic in network processing. The goal of consensus is to design a local (not global) information exchanging strategy, by which all nodes will tend to the same state asymptotically. The fundamental theory of linear consensus roots in the classic linear dynamical system and the Markov chain in stochastic process. Wang et al. gave a necessary and a sufficient condition on the consensus of a general output-feedback linear multi-agent system [10]. The consensus of network was discussed in [11] with infinite time-delay, packet loss and quantization in communication. The model predictive controller was introduced to accelerate the consensus rate [12]. Su et al. investigated the consensus of a distributed T-Z fuzzy filter with time-varying delays and link failures [13]. Ren et al. [14] and Olfati-Saber et al. [15] surveyed a lot of mainstream consensus algorithms, and analyzed the convergence results under various information exchanging strategies.

By their styles of information exchanging, existing consensus algorithms can be divided into three categories as follows,

Style I : Pair-wise exchange;
Style II : Local fusion;
Style III : Asynchronous broadcast gossip.

In Style I, two neighboring nodes exchange and mix their data each time. The pioneering works of Style I brought in the P2P file sharing over the Internet [16].

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http://dx.doi.org/10.1016/j.sigpro.2013.11.017

As for Style II, each node acquires information from all its neighbors, and then makes linear mixing with its own data simultaneously [17]. Style III is of an asynchronous broadcast way, where each node broadcasts data to its neighbors. If the data from the neighbors is received successfully, the node mixes the received data with its own data. Obviously, Style III is very suitable for a wireless network, due to the inherent broadcast property of wireless communication. However, research works on Style III are not as plenary as that on Styles I and II, in which [18,19] are two representative works.

A consensus result can be reached in a deterministic or random way. Most algorithms collected in [15] rely on deterministic information exchanges, where all the nodes will reach a consensus asymptotically as the following definition.

Definition 1. In a network consisting of N nodes, where node i has an initial value at slot 0 as $\boldsymbol{x}_i^0 \in \mathbb{R}^M$, $i = 1, 2, \ldots, N$. The network reaching an asymptotical consensus result means that there exists $\boldsymbol{x}^\infty \in \mathbb{R}^M$ such that $\lim_{k\to\infty}\boldsymbol{x}_i^k = \boldsymbol{x}^\infty$, $i = 1, 2, \ldots, N$. Further, if $\boldsymbol{x}^\infty = \overline{\boldsymbol{x}}^0 \triangleq N^{-1}\sum_i \boldsymbol{x}_i^0$, we have the average consensus (AC) result.

In contrast to deterministic ones, the works of [18–21] investigated the random algorithms involving random information exchanges over the network. For instance as in [19], each node does random broadcast gossip as Style III. Compared with deterministic ones, random algorithms may speed up the convergence rate, and substantially reduce the communication cost [22]. A random consensus algorithm usually guides all the nodes' values in a network to be the same almost surely (a.s.) asymptotically as the following definition.

Definition 2. In a network consisting of N nodes, where node i has an initial value at slot 0 as $\boldsymbol{x}_i^0 \in \mathbb{R}^M$, $i = 1, 2, \ldots, N$. The network reaching probabilistic consensus (PC) means that there exists $\boldsymbol{x}^\infty \in \mathbb{R}^M$ such that $\lim_{k\to\infty}\boldsymbol{x}_i^k = \boldsymbol{x}^\infty$, $i = 1, 2, \ldots, N$ a.s. Further, if $E[\boldsymbol{x}^\infty] = \overline{\boldsymbol{x}}^0 = N^{-1}\sum_i \boldsymbol{x}_i^0$, we have the unbiased consensus (UC) result.

In a wireless network using shared channels, there are possible collisions on receivers. Collisions may change the topology of a network temporarily, which make the consensus procedure sophisticate. Two methods have been considered to solve the problem of collisions. The first is to avoid any possible collision by designing special consensus strategy. Aysal et al. [19] proposed a random gossip algorithm, where only one node may wake up each time and do broadcast. The authors proved that such algorithm can reach UC, and yet not efficiently, since only one node can broadcast each time. In a practical scenario, we expect that more nodes would broadcast simultaneously, such that data will be disseminated over the network faster. Fagnani et al. [21] designed a random broadcast strategy, the collision broadcast gossip algorithm (CBGA), where each node broadcasts with the same probability. However, the authors only proved the UC of CBGA in specific Abelian Caylay topologies, while their algorithm may not reach UC in other connected networks.

As a matter of fact, there is an inherent relationship between the consensus problem and the distributed optimization problem. As indicated in [23], the consensus problem is a special case of the variance minimization problem over the network, where the objective function is $\min \sum_i \|\boldsymbol{x}_i^k - \overline{\boldsymbol{x}}^k\|^2$. The collaborative optimization over network is an important method to solve practical problems in wired and wireless networks, such as signal processing [24], distributed learning [25], and automatic control [26]. In such cases, the objective function is usually with the form of a sum of multiple components, i.e., $f(\boldsymbol{x}) = \sum_{i=1}^N f_i(\boldsymbol{x})$, where each component belongs to a specific processing node in network. Each node expects to obtain a global optimization result by local information exchanges among neighbors [27], which combines the network communication and the distributed computing together [28].

There are mainly three methods to distribute an optimization problem in the literature. The first one is to add explicit constraints by introducing auxiliary variables, and then solve it by the method of multipliers (MoM) [29, Section 3.4]. A widely applied variant of MoM algorithm is the alternative direction MoM (ADMoM) [29,30], which has been used to solve various distributed problems [3–6,8,9].

The second method is the incremental approach [22,31,32], where each node makes a local gradient descent, then relays the result to another node and repeats the procedure. As not requiring the global activity over network, the incremental approach cannot make concurrent computations on multiple nodes, which leads to a slow convergence rate.

The third method is to integrate the consensus algorithm with local computations. Nedic et al. combined the Style II and Style III consensus with the local gradient descent in [33,34] respectively. Ref. [34] is the closest work to the distributed optimization part of this paper. However, the convergence rate of [34] is rather slow, since only one node may wake up in each slot.

The contributions of this paper are mainly twofold. First, we propose a random broadcast gossip strategy, which is a kind of Style III consensus approach. As in [21], we take into account the possible collisions on the receivers due to the asynchronous behavior. We prove that our algorithm can reach UC in any connected network, which is a significant improvement of [19,21]. Second, we integrate the gradient descent with the random broadcast gossip, by which we obtain a fast distributed optimization method for a sum of convex functions.

The rest of this paper is organized as follows. The problem statement and the algorithms are presented in Section 2. The convergence analysis of proposed algorithms is given in Section 3. Simulations with interpretations can be seen in Section 4. Finally, we conclude this paper with some remarks in Section 5.

2. Problem statement and algorithms

The application scenario considered in this paper is as the following assumption:

(A1) All nodes are working in *half-duplex* mode in a slotted-time shared channel, i.e., each node interchanges the roles as a transmitter or a receiver in slotted time, but cannot transmit and receive in the same slot.

(A2) All nodes are equipped with omni-directional antennas with the same coverage radius, such that a pair of