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Resource depletion under uncertainty: implications for mine depreciation, Hartwick's Rule and national accounting

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Abstract

Lozada's equation [Resource and Energy Economics 17 (1995) 137] for the change in value of a non-autonomous dynamic program is generalized to stochastic control and applied to the depreciation of a competitive mine facing price, reserve and discount rate uncertainty. Mine depreciation includes the costs of these risks, as well as an adjustment to the 'net price' used to value depletion and revisions. The change in value equation also provides the basis for a stochastic version of Hartwick's Rule [American Economic Review 67 (1977) 972] for sustainable consumption, that the risk adjusted value of net investment equal zero. The analysis has implications for the Weitzman [Quarterly Journal of Economics 90 (1976) 156] welfare measure and the stochasic Hamiltonian.

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1. Introduction

Interest in sustainable development and the integration of natural resources into national accounts has stimulated the search for theoretically correct measures of natural resource depletion. Empirical studies (Foy, 1991; Bartelmus et al., 1993; Tengblad, 1993; Smith, 1994; US, 1994; Diaz and Harchaoui, 1997) have experimented with a variety of approaches to measuring the depletion of non-renewable resources. Early theoretical discussions of measuring non-renewable resource depletion include Landefeld and Hines (1985), El Serafy (1989), and Hartwick (1990). Lozada (1995) developed a general equation for the change

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in the value-function of a non-autonomous dynamic program, which he used to derive an equation for mine depreciation. Vincent et al. (1997) derived a special case of Lozada's result in their analysis of Hartwick's Rule in a small resource-exporting economy. Hartwick and Long (1999) also derive Lozada's equation as a part of their analysis of Hartwick's Rule in non-autonomous models. Cairns (2000) derives a version of the Lozada's equation for a model of the competitive mine in which the size of the initial capital investment implies a constraint on the capacity of the mine.

Lozada (1995) applies his equation to the problem of optimal extraction by a competitive mine, which is a non-autonomous problem because the price of the extracted resource is exogenous and changes over time. Lozada's equation assumes perfect foresight so that the future price profile of the extracted resource is known with certainty. However, both theoretical studies of mineral extraction and empirical valuation of mineral deposits emphasize the effects of uncertainty about future prices (Pindyck, 1981; Brennan and Schwartz, 1985; Dias and Rocha, 1999) and the size of the remaining resource stock (Pindyck, 1980) on the extraction decision and on the value of the mine. Crabbé (1982) provided an early summary of these sources of uncertainty and their implications for the theory of non-renewable resource extraction. Thus, it is desirable to have an equation for mine depreciation which takes these important uncertainties into account. This paper generalizes Lozada's results to stochastic non-autonomous control problems and to the theory of the competitive mine under uncertainty. The following section presents the stochastic version of Lozada's general equation for the change in the value function and this result is applied to the competitive mine in Section 3.

Lozada (1995) also points out that his equation implies a general version of Hartwick's (1977) Rule for sustainable development, which has been prominent in the literature on sustainable development. Vellinga and Withagen (1996) and Asheim (1997) provide succinct summaries of this literature. Here again, the presence of important uncertainties make it desirable to have a stochastic version of Hartwick's Rule, which is presented in Section 4. Of course, a stochastic Hartwick's Rule is only one of many ways to incorporate uncertainties into the discussion of sustainable development.

Finally, the envelope equation for the current value Hamiltonian, which provides the basis for Lozada's result, also provides the basis for the theory of national accounting. In Section 5, the implications of the stochastic version of the envelope equation for the theory of national accounting, first considered by Aronsson and Löfgren (1995), are discussed.

2. The change in the value function of a non-autonomous stochastic control problem

One formulation of the stochastic control problem (see Fleming and Rishel, 1975; or Fleming and Soner, 1992) may be stated as

$$\underset{\boldsymbol{u}(s)}{\text{maximize}}\, E_{\text{t}}\left[\int_t^T f(\boldsymbol{x}(s), \boldsymbol{u}(s), s)\, \mathrm{d}s + F(\boldsymbol{x}(T))\right] \tag{1}$$

subject to

$$\mathrm{d}\boldsymbol{x}(s) = \boldsymbol{g}(\boldsymbol{x}(s), \boldsymbol{u}(s), s)\, \mathrm{d}s + \boldsymbol{\sigma}(\boldsymbol{x}(s), \boldsymbol{u}(s), s)\, \mathrm{d}\boldsymbol{z}(s) \tag{2}$$