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Discrete Optimization

Fast heuristics for the frequency channel assignment problem in multi-hop wireless networks

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ABSTRACT

Communication links connect pairs of wireless nodes in a wireless network. Links can interfere with each other due to their proximity and transmission power if they use the same frequency channel. Given that a frequency channel is the most important and scarce resource in a wireless network, we wish to minimize the total number of different frequency channels used. We can assign the same channel to multiple different links if the assignment is done in a way that avoids co-channel interference. Given a conflict graph which shows conflicts between pairs of links if they are assigned the same frequency channel, assigning channels to links can be cast as a minimum coloring problem. However the coloring problem is complicated by the fact that acceptably small levels of interference between pairs of links using the same channel can accumulate to cause an unacceptable level of total interference at a given link. In this paper we develop fast and effective methods for frequency channel assignment in multi-hop wireless networks via new heuristics for solving this extended coloring problem. The heuristics are orders of magnitude faster than an exact solution method while consistently returning near-optimum results.

1. Introduction

It is desirable in wireless networks to use the smallest possible number of frequency channels for inter-device communication. Typically a given device will create a communication *link* with a neighboring node by transmitting on a particular frequency channel; this channel can be reused for other links provided it does not cause interference on any other link using the same channel due to its transmission power and proximity. This gives rise to the *channel assignment problem* of assigning frequency channels to communication links so as to minimize the total number of distinct channels used while avoiding interference. Solutions for this problem should be found quickly since wireless networks can change relatively quickly (wireless devices may join or leave the network at any time).

Given the location of the communicating wireless devices, a *conflict graph* can be created in various ways in which nodes represent the communication links between pairs of devices and undirected arcs connect nodes that will experience an unacceptable level of interference if assigned to the same frequency channel. The channel assignment problem requires the assignment of a channel to each node in the conflict graph so that the total number of distinct channels is minimized but no arc connects two nodes having the same channel. This is a standard minimum coloring problem in which colors represent frequency channels. The minimum coloring problem is well known to be NP-hard for general graphs (Karp, 1972).

However there is a further complication: "no interference" means that the amount of interference between two links using the same channel is below a certain threshold, but it is usually not zero. Hence when there are multiple links using the same frequency channel, the cumulative interference may be enough to cause an unacceptable level of interference on some link using that channel, even though the interference on each pair of links is below the tolerance. This complicates the coloring problem.

This paper addresses the frequency channel assignment problem in multi-hop wireless networks by developing fast heuristics for solving this extended coloring problem. We use randomized heuristic algorithms which find a sequence of *weighted maximal independent sets* (WMaISs) in the conflict graph and assign the same channel to all of the nodes in each WMaIS, while considering the cumulative interference constraint. We compare our results to an exact *quadratically constrained binary programming* (QCBP) formulation. Our heuristics return very good results (requiring at most two channels more than the true optimum solution) while being

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several orders of magnitude faster than the exact QCBP formulation. Preliminary work in this regard appears in Chaudhry, Hafez, and Chinneck (2015).

Exact methods guarantee an optimum solution for minimum coloring of the conflict graph but can require an excessive amount of computation time, whereas heuristics can find reasonable solutions much more quickly. Several greedy heuristics for the classical minimum coloring problem have been proposed in the literature. Some well-known examples include *SEQ* (Kucera, 1991), *DSATUR* (Brelaz, 1979), *RLF* (Leighton, 1979), and *IGA* (Culberson & Luo, 1996). *SEQ* is a very simple and commonly used greedy heuristic coloring algorithm. It takes each vertex of the graph sequentially and assigns to it the smallest indexed color that does not appear in one of its neighbors. *DSATUR* arranges the vertices in the decreasing order of degrees. A vertex with highest degree is colored with color 1. A vertex with highest saturation degree is chosen (ties are broken randomly) where the saturation degree of a vertex is defined as the number of different colors to which it is adjacent. The chosen vertex is colored with the lowest numbered color with which it does not conflict. The algorithm stops when all the vertices are colored.

RLF (Recursive Largest First) colors the vertices one class at a time using the following greedy process. Let C be the color class to be constructed which is initially empty. Let V' be the set of uncolored vertices that can be placed in C; V' initially contains all the currently uncolored vertices. Let U be the set of uncolored vertices that cannot be placed in C; U is initially empty. Choose the first vertex x_o from V' that has the maximum number of adjacent vertices in V'. Place x_o in C and move all the vertices in V' that are adjacent to x_o from V' to U. While V' remains nonempty: choose the first vertex x in V' that has the maximum number of adjacent vertices in U; add x to C and move all the vertices in V' that are adjacent to x from V' to U. Considering only vertices in $V\backslash C$, it constructs the next independent set as the second color class and so on.

IGA (Iterated Greedy Algorithm) iteratively colors the graph using the *SEQ* algorithm. At each iteration, the order of the color classes is changed according to some criteria which may lead to a reduction in the number of colors used. The various reordering heuristics include reversing the order of the color classes, placing the color classes in a random order, placing the classes with largest cardinality first or largest fit, placing the smallest cardinality classes first or smallest fit, placing the classes in increasing order by degree sum of the group, and placing the classes in decreasing order by degree sum of the group. The authors used a probabilistic choice of the reordering move to be performed. A ratio of 50:50:30 (largest first:reverse:random) was shown to be efficient. The algorithm in Bollobas and Thomason (1985) selects the maximum independent set from the set of uncolored vertices. The vertices of the selected set are assigned to a new color. This process is repeated until the whole graph is colored. A greedy randomized adaptive search procedure (Feo & Resende, 1989; Feo & Resende, 1995) (*GRASP*) was used in Laguna and Marti (2001) for coloring sparse graphs. It consists of two phases: construction phase and improvement phase. The construction phase uses a randomized version of *RLF* to generate initial colorings. The improvement phase applies a local search procedure to the initial solution in the hope of finding an improvement. *GRASP* is an iterative technique in which each iteration provides a solution. The incumbent solution over all *GRASP* iterations is the final result.

Our heuristic solutions belong to this class of constructive greedy minimum coloring heuristics, but are modified to handle the cumulative conflict in the wireless channel assignment problem. A quick solution is required due to the dynamic nature of wireless networks, so heuristic methods must be employed, but no existing heuristic for solving the minimum coloring stage of the wireless network channel assignment problem considers cumulative interference as is done in this paper (and in Chaudhry et al. (2015)).

An overview of the solution techniques for frequency assignment problems in infrastructure-based (single-hop) wireless networks is provided in Aardal, van Hoesel, Koster, Mannino, and Sassano (2007). Our heuristic and exact solution methods, on the other hand, deal with frequency channel assignment in multi-hop wireless networks, specifically wireless mesh networks. The models in Aardal et al. (2007) assume that each antenna is represented by a vertex in the interference graph whereas each vertex in our conflict graph represents a wireless link between a pair of wireless mesh nodes. The models in Aardal et al. (2007) deal with the requirement of assigning multiple frequency channels to the antenna of a wireless node whereas our heuristic and exact solution methods assign a single frequency channel to the half-duplex wireless link between a pair of wireless mesh nodes. It is also mentioned in Aardal et al. (2007) that formulations in which only one frequency is assigned to a vertex in the interference graph are not considered since they lead to non-linear programs which are hard to solve.

Given the potential locations of cellular base stations, demand nodes representing areas containing certain number of call requests per time unit, number of available frequency channels, coverage and interference information of base stations and channels, Akella, Batta, Sudit, Rogerson, and Blatt (2008) deal with the problem of optimally locating the given number of base stations to maximally cover the demand nodes subject to interference constraints. A simpler case of this problem is considered where a base station can interfere with at most two other bases and the degree of the interference graph is limited to two. A heuristic solution based on simulated annealing is developed to solve this problem. Instead of constructing an interference graph based on a radio propagation model, the conflicts in the interference graph are randomly generated. The number of available frequency channels is assumed to be 41, 61, and 71 to generate results for three different problem sizes.

The problem of configuring the power levels of transmitters to provide service coverage to a set of receivers is considered in D'Andreagiovanni, Mannino, and Sassano (2011, 2013) such that a certain network utility, e.g. number of customers, or expected traffic demand, etc., is maximized. A 0–1 formulation of the problem is presented in D'Andreagiovanni et al. (2011) and a technique to identify a subset of the violated constraints is developed. The interference is approximated by assuming the desired signal to be received at the receiver if it is stronger than its strongest interferer. To reduce the numerical problems produced when transmitter powers are represented by continuous variables, a 0–1 formulation for the problem is proposed in D'Andreagiovanni et al. (2013) by considering a finite set of values for the power levels of transmitters. The SINR (signal-to-interference-plus-noise ratio) model is used to model interference, but the formulation is based on a special case where the desired signal at the receiver has interference from a single transmitter. An algorithm is developed that uses heuristics to search for violated constraints. These formulations (D'Andreagiovanni et al., 2011, 2013) are extended to include different frequency channels (D'Andreagiovanni & Mannino, 2009) in order to evaluate the performance of the proposed solutions in a WiMAX network scenario. Three 7 megahertz or six 3.5 megahertz frequency channels in the 3.4−3.6 gigahertz band are considered to generate experimental results.

The frequency assignment problem of minimizing a cost function based on SIRs (signal-to-interference ratios) at points where reception is required is considered in Graham, Montemanni, Moon, and Smith (2008). Algorithms based on simulated annealing and ant colony system are developed that combine the SIR-based cost function approach with the binary constraint approach to reduce