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Cooperative network design: A Nash bargaining solution approach

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ABSTRACT

The Network Design problem has received increasing attention in recent years. Previous works have addressed this problem considering almost exclusively networks designed by selfish users, which can be consistently suboptimal. This paper addresses the network design issue using cooperative game theory, which permits to study ways to enforce and sustain cooperation among users. Both the Nash bargaining solution and the Shapley value are widely applicable concepts for solving these games. However, the Shapley value presents several drawbacks in this context.

For this reason, we solve the cooperative network design game using the Nash bargaining solution (NBS) concept. More specifically, we extend the NBS approach to the case of multiple players and give an explicit expression for users' cost allocations. We further provide a distributed algorithm for computing the Nash bargaining solution. Then, we compare the NBS to the Shapley value and the Nash equilibrium solution in several network scenarios, including real ISP topologies, showing its advantages and appealing properties in terms of cost allocation to users and computation time to obtain the solution.

Numerical results demonstrate that the proposed Nash bargaining solution approach permits to allocate costs fairly to users in a reasonable computation time, thus representing a very effective framework for the design of efficient and stable networks.

1. Introduction

The Network Design (ND) problem has become increasingly important given the continued growth of computer networks such as the Internet. The design of such networks is generally carried out by a large number of self-interested actors (users, Internet Service Providers …), all of whom seek to optimize the quality and cost of their own operation. In general, for the ND problem we are given a directed graph, where each edge has a nonnegative cost, and a set of players. Each player is identified with a source–destination pair and wants to connect his source to the destination node with the minimum possible cost. Over the past years, the network design problem has been tackled almost exclusively from a non-cooperative point of view. Recent works [1–6] have modeled how selfish agents can build or maintain a large network by paying for possible edges.

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Nash equilibria in such games, however, can be much more expensive than the optimal, centralized solution. This is mainly due to the lack of cooperation among network users, which leads to the design of costly networks.

The underlying assumption in all the above works is that agents are completely non-cooperative entities. However, this assumption could be not entirely realistic, for example when network design involves long-term decisions (e.g., in the case of Autonomous Systems peering relations). It is more natural that agents will discuss possible strategies and, as in other economic markets, form coalitions taking strategic actions that are beneficial to all members of the group. Moreover, incentives could be introduced by some external authority (e.g., the network administrator, government authority) in order to increase the users' cooperation level.

Preliminary works, like [7,8], tried to overcome this limitation by incorporating a socially-aware component in the users' utility functions. This solution, though, can be insufficient to obtain cost-efficient networks in all scenarios. In fact, it has been demonstrated in [8] that, quite surprisingly, highly socially-aware users can form stable networks that are much more expensive than the networks designed by purely selfish users.

To address the above issues, in this paper we first formulate the network design problem as a *cooperative* game, where groups of players (named *coalitions*) coordinate their actions and pool their winnings; consequently, one of the problems is how to divide the cost savings among the members of the formed coalition.

Then, we propose a *Nash bargaining approach*[1] to solve the cooperative network design problem. The Nash bargaining solution (NBS) is a very effective tool to model interactions among negotiators, and is unique for bargaining games satisfying Pareto optimality, symmetry, scale independence, and independence of irrelevant alternatives [9,10]. More specifically, as a key contribution, we extend the Nash bargaining solution for the cooperative network design problem to the case of multiple players with linear constraints, and give explicit expressions for users' cost allocations, assuming that the disagreement point corresponds to players' costs at Nash equilibrium (the cost for players to connect their source–destination nodes in a purely non-cooperative game). To the best of our knowledge, the derived explicit expressions are new. Our other major contribution, in fact, is the demonstration that our proposed cooperative game theory approach, based on the Nash Bargaining Solution, can be actually applied to large networks.

To complement our study, we further focus on the *Shapley value* concept, which is a widely applied solution for cooperative games, since it provides a unique and fair solution [11]. To compute the Shapley value of the cooperative ND game, we consider in this paper three different (natural) definitions for the characteristic function, which associates with every coalition (a subset of players) a real value representing the cost for the coalition. However, we show that the Shapley value presents several limitations in our context: (1) it is non-trivial to define meaningful characteristic functions, (2) the cost allocation determined by the Shapley value can be, in some cases and for some players, even costlier than that obtained at some Nash equilibrium, and (3) for our network design game, it cannot be determined in a reasonable computation time, even when approximation techniques (like that proposed in [12]) are applied.

Finally, we provide a distributed algorithm for computing the Nash bargaining solution. Furthermore, we perform a thorough comparison of the proposed Nash bargaining solution with other classic approaches like the Shapley value and the Nash equilibrium solutions, using different, large-scale network scenarios, including real Internet Service Provider (ISP) topologies. Both exact and approximate methods for computing the Shapley value are considered and compared to our approach.

Numerical results demonstrate that our Nash bargaining solution can provide efficient cost allocations in a short computing time, thus representing a very effective tool to plan efficient and stable networks.

The main contributions of this work can therefore be summarized as follows:

- the formulation of the network design problem as a cooperative game, where players cooperate when connecting their source–destination pairs to reduce their costs.
- The proposition of a novel Nash bargaining solution for the *n*-person cooperative network design problem, which has appealing properties in terms of planning efficient networks and cost allocations in a short computation time.
- The proposition of three definitions for the characteristic function and computation of the Shapley value for the cooperative network design game, showing that this solution requires a long computation time for solving large-scale networks even when sampling-based approximation techniques are used.
- The construction of a distributed algorithm for computing the Nash bargaining solution.
- A thorough comparison of the proposed approach with classic solutions, viz. the Shapley value and the Nash equilibrium concepts, in several realistic and large-size network scenarios, including real ISP topologies.

The paper is organized as follows: Section 2 discusses related work. Section 3 introduces the cooperative network design game, while Section 4 illustrates the proposed Nash bargaining solution along with a distributed approach we propose for its computation. Application scenarios are discussed in Section 5. Section 6 presents numerical results that demonstrate the effectiveness of the NBS approach in several realistic network scenarios, including real ISP topologies. Finally, Section 7 concludes this paper.

[1] The Nash bargaining approach studies situations where two or more agents need to select one of the many possible outcomes of a joint collaboration [9,10]. Each party in the negotiation has the option of leaving the table, in which case the bargaining will result in a disagreement outcome.