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A branch-and-cut algorithm for two-level survivable network design problems

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ABSTRACT

This paper approaches the problem of designing a two-level network protected against single-edge failures. The problem simultaneously decides on the partition of the set of nodes into terminals and hubs, the connection of the hubs through a backbone network (first network level), and the assignment of terminals to hubs and their connection through access networks (second network level). We consider two survivable structures in both network levels. One structure is a two-edge connected network, and the other structure is a ring. There is a limit on the number of nodes in each access network, and there are fixed costs associated with the hubs and the access and backbone links. The aim of the problem is to minimize the total cost. We give integer programming formulations and valid inequalities for the different versions of the problem, solve them using a branch-and-cut algorithm, and discuss computational results. Some of the new inequalities can be used also to solve other problems in the literature, like the plant cycle location problem and the hub location routing problem.

1. Introduction

In this paper, we study several two-level network design problems with survivability requirements in both levels. Telecommunication networks are usually multilayer hierarchical networks where the traffic from different origins are collected and sent to upper levels to be routed towards their destinations. In a typical two-level network, the upper level is called the *backbone network*, and connects the hubs (concentrators, switches, multiplexers) among themselves. The lower level networks are called *access networks* and they connect the users to hubs. Klincewicz [18] uses the notation "backbone structure/access structure" to specify the structure of a two-level network. For instance in a "fully connected/ring network", the backbone network is a complete graph between the hubs, and the access networks are rings, each visiting a subset of users and one hub.

Network survivability, which is the ability of a network to continue functioning in the case of failures, is one of the most critical issues in the design of telecommunications networks. A common assumption is that at most one edge can fail at a time in a network. To ensure survivability in case of single edge failures, the most common topology used is a ring. A ring is a special case of a 2-edge connected subgraph where each node has degree two. A 2-edge connected subgraph (2EC) provides the same level of survivability as a ring in case of edge failures, and may result in less redundant capacity reservation (see, e.g., Karaşan et al. [15] and Shi and Fonseka [29]). We consider these two topologies in the design of a two-level network with protection against a single edge failure. As a result, we study the design problems associated with four different networks: 2EC/2EC, 2EC/ring, ring/2EC and ring/ring networks. We will denote all these problems with the general term of 2-level survivable network design problem (2-LSNDP).

In a 2-LSNDP we are given a set of nodes. The cost of connecting a pair of nodes by a link in the backbone or in an access network is known. There is also a cost associated to select a node as hub. The number of nodes in each access network is limited by the capacity of the hubs, which is a priori given. The problem consists of choosing the nodes to act as hubs and connecting them through a backbone network, and of assigning the non-hub nodes to the hubs and connecting them through access networks, respecting the capacity and the topology requirements. The objective is to minimize the total cost of the resulting two-level network. Fig. 1 shows a ring/ring 2-LSNDP optimal solution for an instance with 15 nodes where the number of nodes in each access network is limited to 3. The solid lines represent the backbone network and the dashed lines represent the access networks. The nodes in the backbone network are the hubs. Fig. 2 shows the

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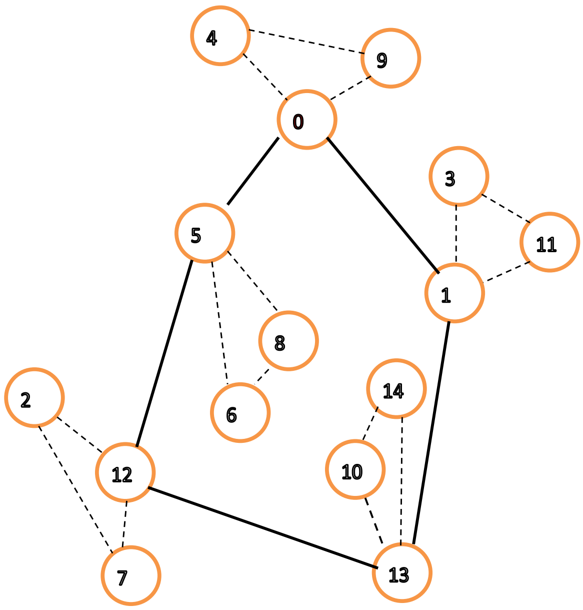

Fig. 1. A ring/ring solution example.

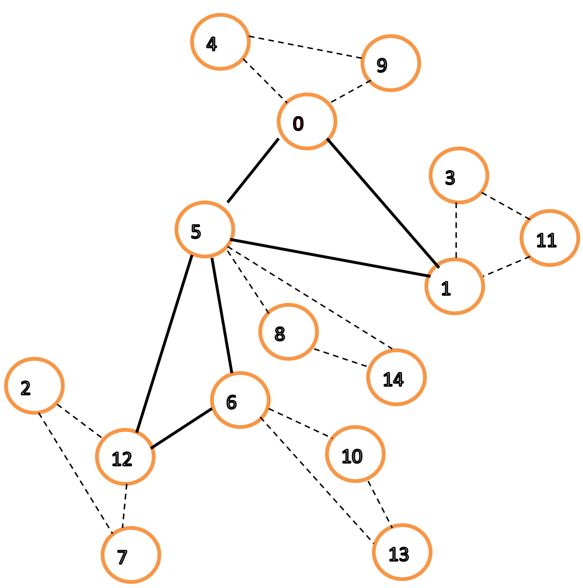

Fig. 2. A 2EC/2EC solution example.

optimal solution for the same instance when the required network structure is 2EC/2EC.

Even though survivability is critical for service providers, there are few studies on designing hierarchical survivable networks. Most studies on survivable network design problems consider a single layer of the network. For reviews of these studies, one can refer to Grötschel et al. [13] and Kerivin and Mahjoub [16]. Polynomially solvable special cases of the survivable network design problem are studied in Kerivin and Mahjoub [17]. The most common network structure in this field is a 2EC subgraph (see, e.g., [23,24,31,33]). Problems related with designing rings of bounded size are studied by Fortz and Labbé [7] and Fortz et al. [8–10]. Generalizations of 2EC networks are studied by Magnanti and Raghavan [22] and Balakrishnan et al. [2].

There are also studies on two-level networks with survivability requirements on the backbone network. For example, Labbé et al. [20] propose a branch-and-cut algorithm for designing a ring as backbone network, while the access networks are direct connections from users to a hub (i.e., star structure). Baldacci et al. [6] address a more general problem where the backbone network allows m rings instead of a single one. Fouilhoux et al. [11] study the variant where the ring structure is replaced by a 2EC network. In all these studies the access networks are forced to be star structures.

The studies that consider survivability at both layers of the network are few. Lee and Koh [21] study the ring/chain network design problem with dual homing where the ring topology in the backbone network is given. They study the design of the access networks. They show that the problem is NP-hard, propose an integer programming formulation and describe a tabu search heuristic. Thomadsen and Stidsen [32] study the ring/ring network design problem. They suggest to solve the design problems associated with different levels sequentially. They propose a branch-and-price algorithm for this purpose. Carroll and McGarraghy [3] also propose to decompose the problems of designing the rings in different levels. Shi and Fonseka [28] study the design of hierarchical self healing rings and propose a heuristic. Proestki and Sinclair [26] and Shi and Fonseka [29] propose heuristic algorithms for the problem with dual homing. Balakrishnan et al. [1] study a generalization of the two-level survivable network design problem to that of multitiers. They analyze worst-case performances of some heuristics for some special cases. Park et al. [25] study a node clustering problem with survivability requirements. Karasan et al. [15] propose a branch-and-cut algorithm for the problem where the backbone network is 2EC and each user node is connected directly to two distinct hub nodes. More recently, Hill and Voß [14] introduce the capacitated ring tree problem where nodes are connected with rings and trees, and rings intersect at a distributor node.

For further related studies, we refer the readers to the following surveys. Klincewicz [18] reviews design problems that involve location of hubs. Gourdin et al. [12] survey location problems encountered in telecommunications network design. Soriano et al. [30] provide an overview of design and dimensioning problems in survivable SDH/SONET networks. Studies on combined location and network design problems are reviewed by Contreras and Fernández [4].

In summary, there are few studies that consider the design of two-level networks with survivability requirements in both levels. Most of such studies are on designing ring/ring networks and most of the proposed approaches are of heuristic nature. The contribution of this paper is to propose strong formulations and exact solution methods for the two-level survivable network design problem where both rings and 2-edge connected networks are used to ensure survivability.

The remainder of the paper is organized as follows. The mathematical model for the different variants of the problem is given in Section 2. Section 3 presents several families of valid inequalities to strengthen the linear-programming relaxations. Section 4 details a branch-and-cut approach based on the formulation and inequalities presented in the previous sections. The performance of this approach is analyzed in Section 5 on a large collection of instances. Finally, the paper ends with conclusions in Section 6.

2. MIP models

We first introduce the notations. Let $V = \{0, 1, \dots, n-1\}$ be the set of nodes, where node 0 stands for the root and is considered to be a hub. Note that this is not a restrictive assumption as one can solve the problem for different root nodes if one has not been a priori fixed. Let $E = \{\{i,j\} : i,j \in V, i < j\}$ be the set of potential links. We assume $G = (V, E)$ to be an undirected graph and we do not