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On the present worth of multivariate quality loss

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Abstract

Quality loss function has been introduced, by Taguchi, to be a quality performance measure for products since the 1980s. In this paper, we extend the work of Teran et al. [The Engineering Economist 42 (1) (1996) 39–52] and incorporate the concept of time value of money into the multivariate loss function. First, the model for the present worth of the expected multivariate quality loss (PWML) is established and its solution procedure is developed. Then, an example is provided to illustrate how the model can be applied. Some sensitivity analyses are conducted to study the effects of planning horizon, customer discount rate and coefficients of parameter drift on the optimal means at production time and the associated quality loss. From the results of analysis, the longer the planning horizon of the product is, the farther the means should be set relative to the targets at production time. Also, as the customer discount rate increases, the mean should be set closer to the target at production time. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Manufacturing and service industries have experienced increasing demand on quality since the latter part of 1970s. The most widely accepted definition of quality is "fitness for use" [1]. This definition implies that the customer is the true evaluator of product quality. Various quality performance measures, such as percent defective and process capability index Cp, are commonly used to quantitatively evaluate product quality. However, the percent defective of products in shipped goods is generally small. Even in developing countries, a manufacturer usually screens products to ensure that only non-defective units are shipped. When defective product units are not shipped, the customers are not directly affected except in increased cost. Thus, the occurrence of defective units should be considered both a quality and a cost problem [2]. Another commonly used measure of quality level for shipped products is the process capability index Cp, which is defined as

$$\mathrm{Cp} = \frac{\mathrm{USL} - \mathrm{LSL}}{6\sigma}, \tag{1}$$

where LSL and USL are the lower and upper specification limits of the quality characteristic and σ denotes its process standard deviation. There are many manufacturers in the USA and Japan who require their suppliers to produce units with a Cp of more than one [2]. However, the major

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Nomenclature

$\mu_i(t)$	the mean of the ith characteristic at time t
$\sigma_i^2(t)$	the variance of the ith characteristic at time t
$\sigma_{ij}(t)$	the covariance of the ith and jth characteristics at time t
$d_i(t)$	the bias of the ith characteristic at time t
M_i	the target value of the ith characteristic
T	planning horizon of the product
r	the product user's discount rate
PWML	present worth of expected multivariate quality loss
a_i	the linear drift rate on mean per time unit for the ith characteristic
b_i	the quadratic drift rate on mean per time unit for the ith characteristic
c_i	the increasing rate of variance per time unit for the ith characteristic
h_{ij}	the changing rate for the covariance of the ith and jth characteristics per time unit

disadvantages of Cp are that there is no apparent immediate basis for its optimal value and that the managers and engineers cannot comprehend the significance of its values; e.g., what is the actual quality improvement (QI) when Cp changes from 0.95 to 1.05? Based on these reasons, Taguchi [3] introduced a monetary evaluation, called loss function, to evaluate the quality of a product. According to Taguchi [3], quality loss is always incurred when a product's functional quality characteristic (denoted by y) deviates from its ideal target (or nominal values), denoted by M, regardless of how small the deviation is. The loss function for a product unit is defined as

$$L(y) = k(y - M)^2, \tag{2}$$

where k is a proportionality constant depending on the societal losses at the specification limits and the width of the specification. Consequently, the expected value of quality loss for a characteristic y (obtained by applying the expectation operator to Eq. (2)) is given by

$$E[L(y)] = k[\sigma^2 + (\mu - M)^2] \tag{3}$$

where μ is the mean of y and σ^2 denotes its variance. The loss function in Eq. (2) is almost restricted to the shipment time of a product and ignores the degradation effect of usage over time on quality performance.

By considering the effect of product deterioration on quality performance, Teran et al. [4] introduced the present worth of expected quality loss (PWL) as a quality performance measure. They modeled the expected quality loss from the standpoint of a discounted cash flow, which considers the time value of money. Due to this property, economic analysis of quality loss can be done by considering the time the customer incurs it. Use of discounted cash flows presents the advantage of being within a framework that is common in studies related to long-term financial decisions [5]. As a result, the present worth of expected quality loss may serve as a means to perform the translation of quality performance measures into financial measures. The advantages of such a quality measure include:

1. It is expressed in monetary terms, and
2. It incorporates the effect of product degradation.

Using the index of PWL, Teran et al. [4] have shown that, for the nominal-the-best (NTB) type of quality characteristic, the recommendation of the process mean at the nominal value (at production time) is a practice that minimizes quality losses only for cases of non-degradable products. (Note that the NTB type of quality characteristic has an ideal target such that deviations below and above the target are undesired.) The index of PWL will be briefly reviewed in the next section.

The expected value of quality loss in Eq. (3) measures only one quality characteristic y. However, many industrial products and processes are characterized by more than one measurable quantity, and their joint effect describes product quality. For example, in the production of synthetic fiber, the tensile strength and diameter may be equally